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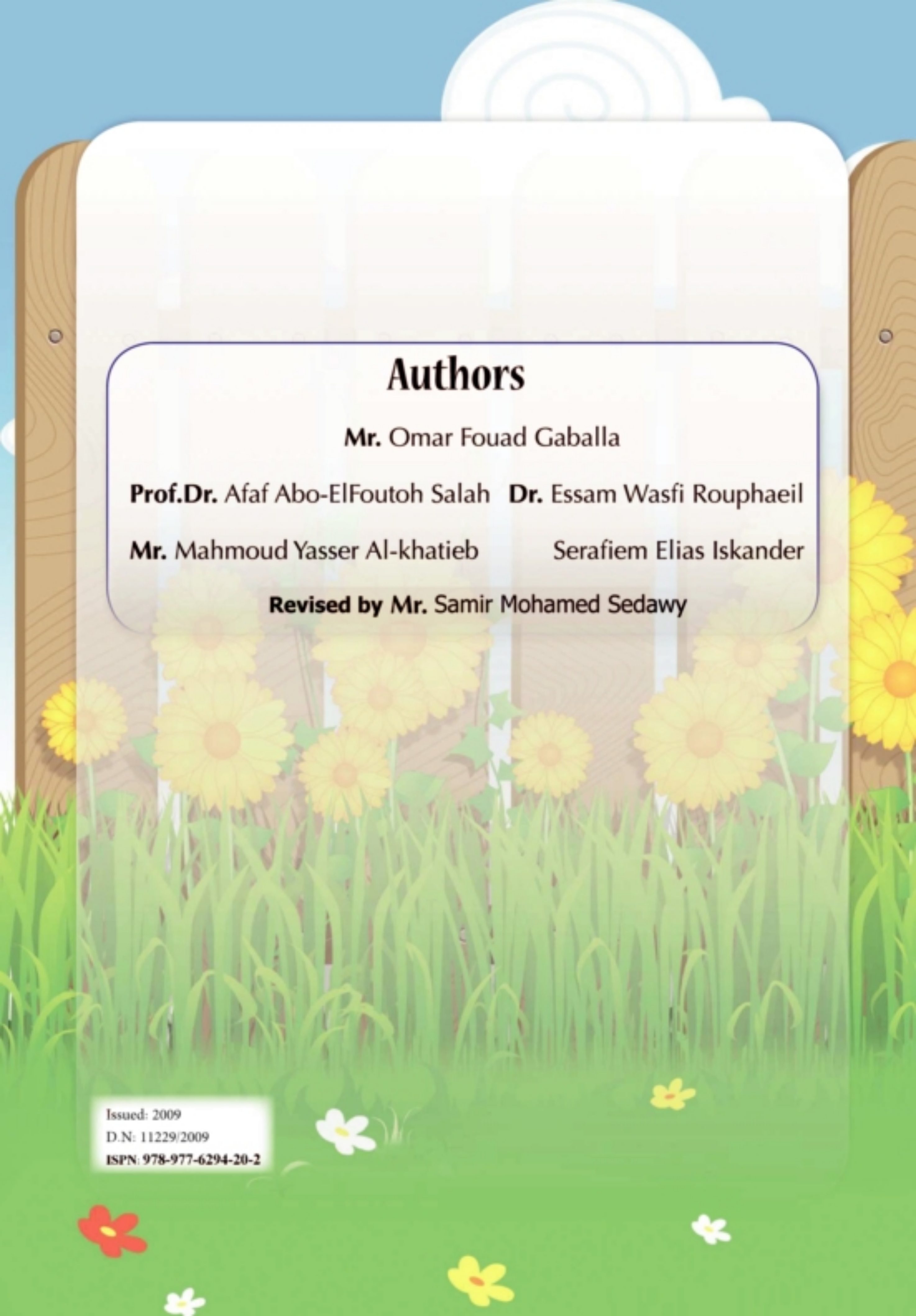
MATHEMATICS

2019 - 2020

غير مصرح بتداول هذا الكتاب خارج
وزارة التربية والتعليم والتعليم الفنى

For Preparatory Year Two
Second Term

Student s Book



Authors

Mr. Omar Fouad Gaballa

Prof.Dr. Afaf Abo-ElFoutoh Salah **Dr.** Essam Wasfi Rouphaeil

Mr. Mahmoud Yasser Al-khatieb Serafiem Elias Iskander

Revised by Mr. Samir Mohamed Sedawy

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بسم الله الرحمن الرحيم

Dear students:

It is extremely great pleasure to introduce the mathematics book for second preparatory. We have been specially cautious to make your learning to the mathematics enjoyable and useful since it has many practical applications in real life as well as in the other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate the mathematicians roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining the patterns of positive thinking which pave your way to creativity .

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration .

Our great interest here is to help you get the information by your self in order to develop your self-study skills.

Calculators and computer sets are used when there's a need for. Exercises, practices, general exams, portfolios, unit test, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

Authors

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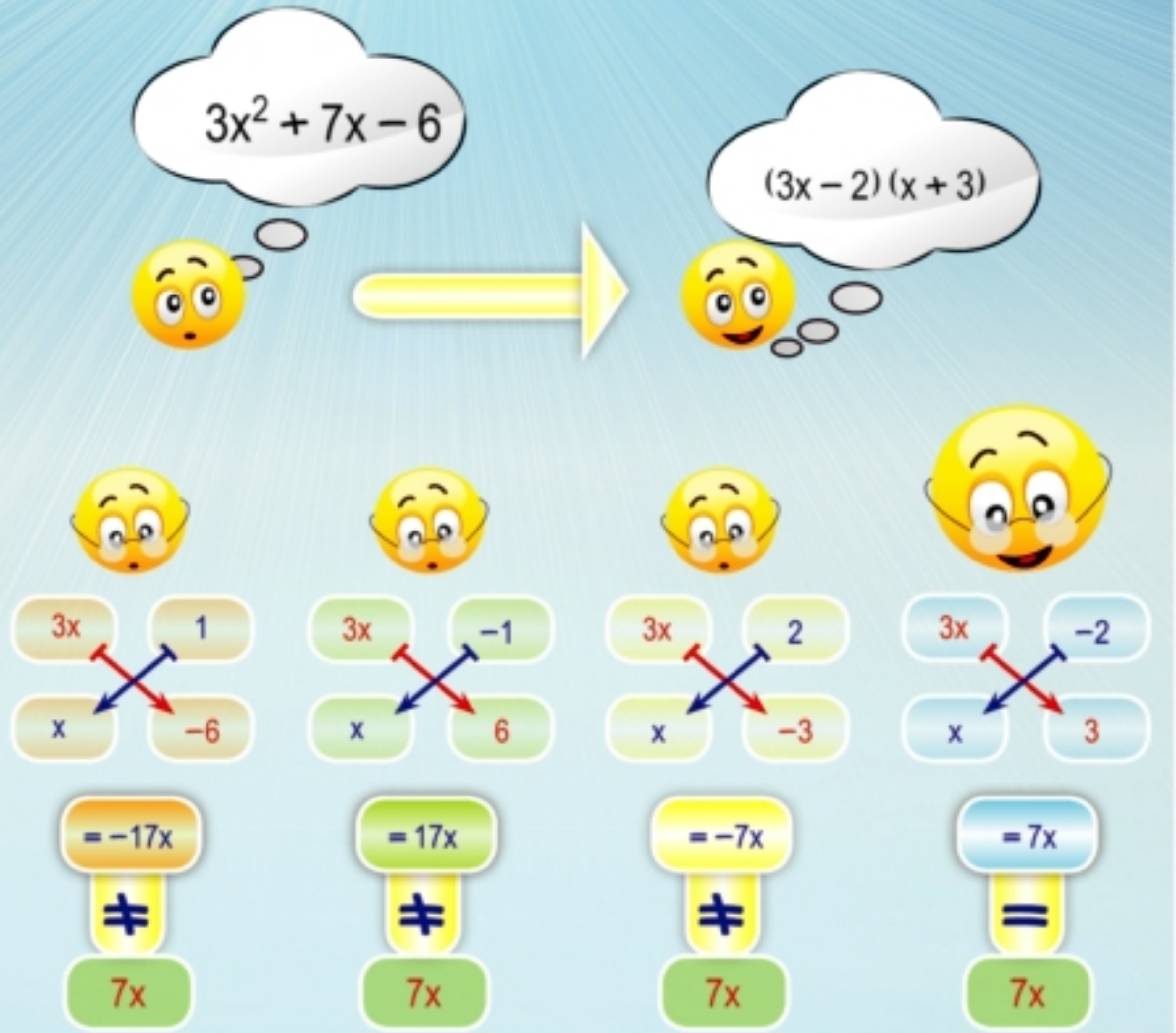
The used Mathematical Symbols

N	The set of natural numbers	$\perp$	perpendicular to
Z	The set of integer numbers	$//$	parallel to
Q	The set of rational numbers	$\overline{AB}$	Line segment AB
Q`	The set of irrational numbers	$\overrightarrow{AB}$	Ray AB
R	The set of real number	$\overleftrightarrow{AB}$	straight line AB
$\sqrt{a}$	Square root of number a	$m(\angle L)$	measure of angle L
$\sqrt[3]{a}$	Cube root of number a	$\sim$	Similarity
$[a, b]$	Closed interval	$<$	less than
$]a, b[$	Open interval	$\leqslant$	less than or equal to
$[a, b[$	Half-open (closed) interval	$>$	Gearter than
$]a,b]$	Half-open (closed) interval	$\geqslant$	Gearter than or equal to
$]-\infty, a]$	Infinte interval	P(E)	probability of occuring event (E)
$\equiv$	congurence		

UNIT ONE

1

FACTORIZATION



Unit One

Lesson One

Factorizing Trinomials

Think and Discuss

You will learn

- The meaning of factorizing an algebraic expression
- factorizing a trinomial

Key-Terms

- Factorizing.
- An algebraic expression
- A trinomial

You have learned that :

Factorizing an integer means to write it as a product of two or more factors.

For example:

$12 = 3 \times 4$, $12 = -3 \times -4$ or $12 = 2 \times 6$ or.........

We have learned before to factorize by taking out the highest common factor H.C.F:

For example:

$6x^2y^2 - 9x^3y = 3x^2y(2y - 3x)$

Practice

Factorize using H.C.F for all the following terms:

1. $2 \times (m + 3) - 4y(m + 3)$
2. $a(a-b) - b(b - a)$

You know that: $(x + 3)(x + 4) = x(x + 4) + 3(x + 4)$

$$= x^2 + 4x + 3x + 3 \times 4$$
$$= x^2 + (4 + 3)x + 12$$
$$= x^2 + 7x + 12$$

The algebraic expression $(X^2 + 7X + 12)$ **is often called a trinomial**.

By using the previous multiplication properties. Can you factorize the expression $(x^2 + 7x + 12)$ into two factors?

First: Factorize x^2 into $x \times x$

Product	sum
1 × 12	13
-12 × -1	-13
2 × 6	8
-2 × -6	-8
3 × 4	7
-3 × -4	-7

Second: Guess and check two numbers whose product is 12 and whose sum is 7. They are 3 and 4. Thus, $x^2 + 7x + 12 = (x + 3)(x + 4)$

Practice

1. Find two numbers whose product is 20 and whose sum is 9
2. Find two numbers whose product is 12 and whose sum is -8
3. Find two numbers whose product is -24 and whose sum is 5
4. Find two numbers whose product is -15 and whose sum is -14

First: Factorizing Quadratic Trinomials in the form $x^2 + bx + c$

Factorize this expression into two linear factors:

- the first term in each factor is x.
- the last two terms are two numbers whose product is C and whose sum is b.

Exemples :

1. **Factorize the expression:** $x^2 - 5x + 6$

Look for two numbers whose product is 6 and whose sum is -5.
They are -2 and -3.

Thus, $x^2 - 5x + 6 = (x - 2)(x - 3)$

2. **Factorize the expression:** $x^2 - 5x - 6$

Look for two numbers whose product is -6 and whose sum is -5. They are 1 and -6.

Thus, $x^2 - 5x - 6 = (x + 1)(x - 6)$

3. **Factorize the expression:** $3y^2 - 48 + 18y$

① The expression should be orderd according to the descending exponenets of y.
The expression will be: $3y^2 + 18y - 48$

② Note that H.C.F for all of the terms then take out H.C.F which is 3.
The expression will be: $3(y^2 + 6y - 16)$

③ Look for two numbers whose product is -16 and whose sum is 6.
They are -2 and 8.

$\therefore$ The expression $= 3(y - 2)(y + 8)$

4 **Factorize the expression:** $m^4 - 6m^2n + 5n^2$

Solution

① m^4 is factorized as $m^2 \times m^2$

② Look for two numbers whose product is $(5n^2)$ and whose sum is $(-6n)$. They are $-n$ and $-5n$.

Thus, the expression $= (m^2 - n)(m^2 - 5n)$

Practice

Factorize each expression of the following:

1. $x^2 + 11x + 10$
2. $x^2 - 7x + 10$
3. $x^2 - 3x - 10$
4. $x^2 - 7x + 12$
5. $x^2 + 4x - 12$
6. $x^2 - x - 12$
7. $x^2 - 8x + 12$
8. $y^2 - 20y + 51$
9. $y^2 - 50y - 51$
10. $5x^2 - 10x - 15$
11. $x^4 - 9x^2 + 20$
12. $x^3 - 3x^2 - 28x$
13. $x^2 - 5xy - 24y^2$
14. $b^2 + 3bc - 10c^2$
15. $-x^2 + 2x + 63$

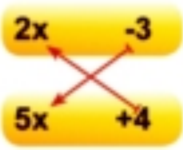
Seconed: Factorizing Quadratic Trinomials of the form $ax^2 + bx + c$, where $a \neq \pm 1$

You know that:

$$(2x - 3)(5x + 4) = \underbrace{10x^2}_{2x \times 5x} + \underbrace{\left(8x + (-15x)\right)}_{\text{Product of inner terms + product of outer terms}} + \underbrace{(-12)}_{-3 \times 4}$$

i.e. $(2x - 3)(5x + 4) = 10x^2 - 7x - 12$

Reverse the process to factorize the quadratic trinomial $10x^2 - 7x - 12$ let's do some trials. The opposite diagram will help you to factorize the expression.



Middle Term $= (2x)(4) + (-3)(5x)$

$= -7x$

$\therefore 10x^2 - 7x - 12 = (2x - 3)(5x + 4)$

Exemple (1)

Factorize the expression $3x^2 + 7x - 6$

Solution

Note that $3x^2 = (3x)(\times)$ while -6 is factorized as follows

1×-6 , -1×6 , 2×-3 or -2×3. Observe the following trials to get a true answer:

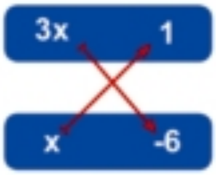


Fig. (1)

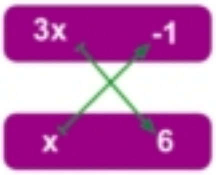


Fig. (2)

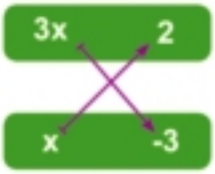


Fig. (3)

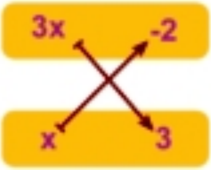


Fig. (4)

In fig. (1): : $3x \times -6 + x \times 1 = -17x \neq$ Middle Term (False).

In fig.(2): : $3x \times 6 + x \times -1 = 17x \neq$ Middle Term (False).

In fig.(3): : $3x \times -3 + x \times 2 = -7x \neq$ Middle Term (False).

In fig. (4): : $3x \times 3 + x \times -2 = 7x =$ Middle Term (True).

$\therefore 3x^2 + 7x - 6 = (3x - 2)(x + 3)$

Exemple (2)

Factorize the expression $15x^4 - 21z^2 - 6x^2z$

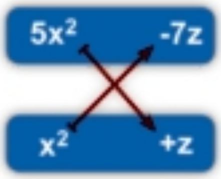
Solution

① The expression after the order is : $15x^4 - 6x^2z - 21z^2$. Note that H.C.F = 3 :

the expression $= 3(5x^4 - 2x^2z - 7z^2)$

② $\because$ The third term is negative

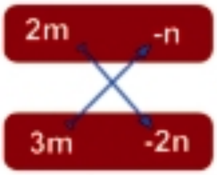
$\therefore$ The Factors of $-7z^2$ have opposite signs.



$\therefore$ The expression $= 3(5x^2 - 7z)(x^2 + z)$

Exemple (3)

Factorize the expression $6m^2 + n(2n - 7m)$



Solution

The given Expression $= 6m^2 + 2n^2 - 7nm$

$= 6m^2 - 7nm + 2n^2 = (2m - n)(3m - 2n)$

Note that: You can check your answer by testing multiplication visually to get the original expression, before the factorize.

Exercices (1 - 1)

First : **Complete:** **the missing terms to get true factorization :**

1. $3x^2 + 7x - 6 = (3x - \ldots\ldots)(\ldots\ldots + \ldots\ldots)$
2. $2x^2 + x - 6 = (\ldots\ldots + \ldots\ldots)(x - \ldots\ldots)$
3. $5x^2 - 2x - 7 = (5x - \ldots\ldots)(x + \ldots\ldots)$
4. $6x^2 - 11x - 10 = (2x - \ldots\ldots)(\ldots\ldots + 2)$
5. $3x^2 + 10x + 8 = (\ldots\ldots + 4)(x + \ldots\ldots)$

Second: **Factorize** **each of the following expressions:**

1. $2x^2 + 3x + 1$
2. $2y^2 + 5y + 3$
3. $3a^2 + 7a + 2$
4. $5z^2 - 7z + 2$
5. $6x^2 - 11x + 3$
6. $3m^2 - 19m + 6$
7. $5x^2 - 3x - 2$
8. $3y^2 + 7y - 6$
9. $2m^2 - 9m - 5$
10. $8z^2 + 2z - 3$
11. $5a^2 - 18a + 16$
12. $8x^3 - 27x^2 - 20x$
13. $6x^2 - 47xy - 63y^2$
14. $10a^2 + 11ab - 18b^2$
15. $3x^2 - 20xy - 7y^2$
16. $7x^4 + 23x^2y - 30y^2$
17. $6a^2 - 19ab - 7b^2$
18. $5y^2 - 4x(7y + 3x)$
19. $18x^5 + 33x^3 - 30x$
20. $25m - 10 + 15m^2$
21. $21x^2y^2 + 6x^2y^3 - 15x^2y^4$
22. $a^2b^2 - 24ab^2 + 143b^2$

Third: The area of a rectangle is $(2x^2 + 19x + 35)$ cm². Find the dimensions of the rectangle in terms of x, then find the perimeter of the rectangle, when $x = 3$.

Unit One

Lesson Two

Factorizing the Perfect-Square Trinomials

Think and Discuss

You will learn

- To factorize perfect-square trinomials

Key-Terms

- a perfect-square.

You have learned before:

$$(2x - 3)^2 = 4x^2 - 12x + 9$$
$$(5y + 7x)^2 = 25y^2 + 70xy + 49x^2$$
$$(L^2 - 5m)^2 = L^4 - 10L^2m + 25m^2$$

Trinomials like $4x^2 - 12x + 9$, $25y^2 + 70xy + 49x^2$, $L^4 - 10L^2m + 25m^2$ **are called perfect-squares.**

Note that

① Each of the first and third terms are perfect squares.

② The middle term = ± 2 × square root of first term × square root of third term.

The factorization of a perfect-square trinomial is written in the form:

Perfect-Square Trinomial =

$$\left(\sqrt{\text{First term}} \;\underset{\substack{\downarrow \\ \text{the sign of the middle term}}}{\pm}\; \sqrt{\text{Third term}}\right)^2$$

Ex. $9x^2 - 30x + 25 = (\sqrt{9x^2} - \sqrt{25})^2 = (3x - 5)^2$

$L^4 + 14L^2m + 49m^2 = (\sqrt{L^4} + \sqrt{49m^2})^2 = (L^2 + 7m)^2$

Note that

① Take out H.C.F, if existed.

② To order the terms descendingly according to the exponent of one variable.

Exemple (١)

Determine which of the following trinomial expressions is a perfect-square, then factorize it in the form of a perfect-square:

A $25x^2 - 30x + 9$ **B** $m^2 + 4m - 4$ **C** $49a^2 + 70ab^2 + 25b^4$

Solution

A $25x^2 = (5x)^2$ and $9 = (3)^2$ the first and third terms are perfect-squares The Middle Term $= 2(5x)(3) = 30$.

$\therefore$ The expression $25x^2 - 30x + 9$ is a perfect square and the expression $= (5x - 3)^2$

B $m^2 + 4m - 4$ is not a perfect-square, where the third term is negative.

C The first term $49a^2 = (7a)^2$ is a perfect-square and the third term $= 25b^4 = (5b^2)^2$ is a perfect-square and the middle term$= 2(7a)(5b^2) = 70ab^2 =$ Middle Term.

$\therefore$ The expression $49a^2 + 70ab^2 + 25b^4$ is a perfect-square trinomial and the expression $= (7a + 5b^2)^2$

Exemple (2)

Complete the missing term to make a perfect-square in each of the following expressions then factorize each expression.

A $4y^2 \pm \ldots\ldots + 121$ **B** $25a^2 - 30ab \ldots.$

Solution

A The middle term $= \pm 2(\sqrt{\text{first term}} \times \sqrt{\text{third term}}) = \pm 2(2y)(11) = \pm 44y$

$\therefore$ The perfect-square trinomial of the expression$= 4y^2 \pm 44y + 121$ and the expression $= (2y \pm 11)^2$

$25a^2 = (5a)^2$

B The middle term $= -30ab = 2(5a) \times$ square root of the third term

The square root of the third term $= \frac{-30ab}{2 \times 5a} = -3b$

The third term $= (-3b)^2 = 9b^2$

$\therefore$ The perfect-square trimnial $= 25a^2 - 30ab + 9b^2$. the expression$= (5a - 3b)^2$

Practice

Complete the missing term in the expression .... $+ 12x^2 + 36$ to make a perfect-square trinomial, then factorize it:

Exemple (3)

Use factorization to evaluate: $(7.3)^2 + 2 \times 7.3 \times 2.7 + (2.7)^2$

Solution

We notice that the numerical expression is in the form of a perfect-square trinomial, so it can be written in the form The expression, $= (7.3 + 2.7)^2 = (10)^2 = 100$

Practice

Use factorization to evaluate: $(574)^2 - 2 \times 574 \times 573 + (573)^2$

Exemple (4)

Factorize each of the following expressions:

A $5x^3 + 50x^2 + 125x$

B $40a^2b - 50a^4 - 8b^2$

Solution

A Take out H.C.F:

$\therefore$ The expression $= 5x(x^2 + 10x + 25) = 5x(x + 5)^2$

B The expression $= 2(20a^2b - 25a^4 - 4b^2)$ descending order according to the exponent of a:

$= -2(25a^4 - 20a^2b + 4b^2)$

$= -2(5a^2 - 2b)^2$

Exercices (1 - 2)

1 Fill in the blanks to get a perfect-square trinomial:

A $4x^2 \ldots\ldots + 1$

B $a^2 - 6ab + \ldots\ldots$

C $\ldots\ldots - 18y^2 + 81$

D $\frac{1}{25}x^2 \ldots\ldots + \frac{1}{4}y^2$

E $25\,m^2 + 20mn \ldots\ldots$

F $z^4 \ldots\ldots + 49\,L^2$

2 Which of the following expressions is a perfect-square? Factorize each perfect-square:

A $x^2 - 12x + 36$

B $25x^2 - 15x + 9$

C $m^2 - 6m - 9$

D $4\,a^2 + 14\,ab + 49\,b^2$

E $0{,}01\,x^2 - 0{,}2\,x + 1$

F $\frac{1}{4}\,y^2 - y + 4$

3 Choose the true answer:

A Let $x^2 + 14x + b$ be a perfect-square, then $b = \ldots\ldots$ (2, 7, 14, 49)

B Let $(x + y)^2 = 64$, $xy = 15$, then $x^2 + y^2 = \ldots\ldots$ (8, 34, -34, 49)

C Let $a^2 + b^2 = 11$, $ab = 5$ then $a - b = \ldots\ldots$ (6a, ± 1, 1 , -1)

D $(99)^2 + 2\,(99) + 1 = \ldots\ldots$ (100, 10000, 410, $(98)^2$)

E Let $a^2 + 2\,ab + b^2 = 25$, then $a + b = \ldots\ldots$ (5, -5, ± 5, 12.5)

F Let $x^2 + k\,x + 25$ be a perfect-square, then $k = \ldots$ (5, 10, ± 10, ±5)

4 Factorize each of the following expressions:

A $m^2 - 2m + 1$

B $9x^2 + 12x + 4$

C $36 - 60\,k + 25\,k^2$

D $4\,x^2 - 4\,xy + y^2$

E $9\,a^2 + 6\,ab + b^2$

F $25\,b^2 - 10\,b + 1$

5 Factorize each of the following each expressions:

A $18\,y^2 - 12y + 2$

B $24x + 24x^2 + 6x^3$

C $6\,a^4 - 12a^2b^2 + 6b^4$

D $4\,b^2c + bc^2 + 4\,b^3$

E $3z + 42\,z^4 + 147z^7$

F $20\,ay^2 - 60ay + 45a$

6 Use factorization to evaluate the value of each of the following:

A $(20{,}7)^2 - 1{,}4 \times 20{,}7 + (0{,}7)^2$

B $(997)^2 + 6 \times 997 + 9$

Factorizing theDifference of two Squares

Unit One
Lesson Theree

Think and Discuss

You will learn

- To factorize the difference of two squares.

Key-Terms

- Difference of two squares

You have learned before:

$$(x + y)(x - y) = x^2 - y^2$$

The algebraic expression $x^2 - y^2$ is called the difference of two squares.

The difference of two square quantities = the sum of the two quantities × the difference of the two quantities.

$$x^2 - y^2 = (x + y)(x - y)$$

Exemple (1)

Factorize each of the following expressions:

A $49x^2 - 25$ B $(2y - 3)^2 - 1$

C $27m^3 - 48mn^6$ D $(x + y)^2 - (x - y)^2$

Solution

A $49x^2 - 25 = (7x + 5)(7x - 5)$

B $(2y - 3)^2 - 1 = [(2y - 3) + 1][(2y - 3) - 1]$
$= (2y - 2)(2y - 4)$
$= 2(y - 1) \times 2(y - 2) = 4(y - 1)(y - 2)$

C $27m^3 - 48mn^6 = 3m(9m^2 - 16n^6)$
$= 3m(3m + 4n^3)(3m - 4n^3)$

D $(x + y)^2 - (x - y)^2 = [(x + y) + (x - y)][(x + y) - (x - y)]$

$= 2x \times 2y = 4xy$

Exemples

2 Use factorization to evaluate the value of each of the following:

A $(763)^2 - (237)^2$ **B** $(999)^2 - 1$

Solution

A The algebraic expression = (763 + 237) (763 - 237) = 1000 × 526 = 526000

B The algebraic expression= (999 + 1) (999 - 1) = 1000 × 998 = 998000

3 Factorize the expression $81\,x^4 - 16\,y^4$

Solution

$$81x^4 - 16y^4 = (9x^2 + 4y^2)(9x^2 - 4y^2)$$
$$= (9x^2 + 4y^2)(3x + 2y)(3x - 2y)$$

Exercices (1-3)

1 Factorize each of the following expressions, if possible:

A $x^2 - 4$ **B** $9 - y^2$ **C** $-9x^2 + 25$

D $8x^2 - 50$ **E** $a^2 - b^2 c^4$ **F** $225\,x^2 - y^2$

G $(x + 1)^2 - (x - 1)^2$ **H** $9(m - 1)^2 - 25\,(m + 1)^2$ **I** $x^{100} - 1$

2 Use factorization to evaluate:

A $(77)^2 - (23)^2$ **B** $(8{,}27)^2 - (1{,}23)^2$ **C** 31×29

D The length of one side of the right angle in a triangle if the length of the hypotenuse is 41cm and the third side is 40cm.

3 If $x^2 - y^2 = 20$ and $x + y = 10$, then find value of $x - y$

4 If $L - m = 9$, $L + m = 15$, then find the value of the expression $L^2 - m^2$

5 If $4x^2 - y^2 = -32$ and $2x + y = 8$, then Find the value of the expression $y - 2x$

6 Factorize each of the following expressions:

A $(x + y + 5)^2 - (x - y - 5)^2$ **B** $(a + b + c)^2 - (a - b - c)^2$

Factorizing the Sum and the Difference of two Cubes

Unit One

Lesson Four

Think and Discuss

First:factorizing the sum of two cubes

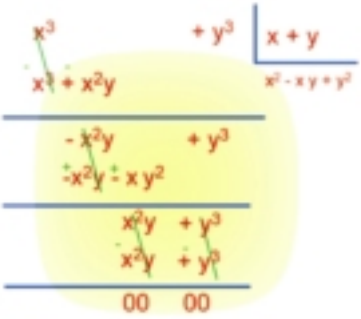
The Teacher asked a student:	Can you factorize the expression : $x^3 + y^3$?
The student answered:	I expect that $(x + y)$ is one of its factors.
The teacher said:	Can you find the other factor in x^3+y^3?
The student answered:	To know the other factor in x^3+y^3 we divide $(x^3 + y^3) \div (x + y)$ using the long division which you have learned previously. The quotient will be $x^2 - x\,y + y^2$

You will learn

- To factorize the sum of two cubes.
- To factorize the difference between two cubes.

Key-Terms

- The sum of two cubes.
- The difference between two cubes.



The algebraic expression $x^3 + y^3$ is called the sum of two cubes **and can be factorized as follows:**

$$x^3 + y^3 = (x + y)(x^2 - x\,y + y^2)$$

Exemple :

$$8x^3 + 27 = (2x)^3 + (3)^3 = (2x + 3)\,[(2x)^2 - 2x \times 3 + (3)^2]$$
$$= (2x + 3)(4x^2 - 6x + 9)$$

Second: factorizing the difference between two cubes:

The algebraic expression $x^3 - y^3$ is called the difference between two cubes **and can be factorized as follows:**

$$x^3 - y^3 = x^3 + (-y)^3$$
$$= \big(x + (-y)\big)\,[x^2 - x\,(-y) + (-y)^2]$$

$$\therefore x^3 - y^3 = (x - y)(x^2 + x\,y + y^2)$$

$$\begin{aligned}125\,a^3 - b^6 &= (5a)^3 - (b^2)^3\\ &= (5a - b^2)(25a^2 + 5\,ab^2 + b^4)\end{aligned}$$

Exemples :

1 **Factorize each of the following expressions:**

A $x^3 + 343y^3$ **B** $40a^3 + 135\,b^3$

C $(x + z)^3 - x^3$ **D** $x^6 - 64\,y^6$

Solution

A
$$\begin{aligned}x^3 + 343\,y^3 &= (x)^3 + (7y)^3\\ &= (x + 7y)(x^2 - 7\,xy + 49\,y^2)\end{aligned}$$

B
$$\begin{aligned}40\,a^3 + 135\,b^3 = 5\,(8a^3 + 27\,b^3) &= 5\,[(2a)^3 + (3\,b)^3]\\ &= 5\,(2a + 3b)\,(\,4\,a^2 - 6\,ab + 9\,b^2)\end{aligned}$$

C
$$\begin{aligned}(x + z)^3 - x^3 &= [(x + z) - x][(x + z)^2 + x(x + z) + x^2]\\ &= z\,(x^2 + 2\,xz + z^2 + x^2 + xz + x^2)\\ &= z\,(3x^2 + 3\,xz + z^2)\end{aligned}$$

D $x^6 - 64\,y^6$

Note that The algebraic expression $x^6 - 64y^6$ can be factorized as a difference between two cubes and as a difference between two squares as well. You must start factorizing it first as a difference between two square, then factorize the resulting factors.

$$\begin{aligned}x^6 - 64y^6 &= (x^3 + 8y^3)(x^3 - 8y^3)\\ &= (x + 2y)(x^2 - 2xy + 4\,y^2)(x - 2y)(x^2 + 2xy + 4\,y^2)\end{aligned}$$

2 Si $x^2 - y^2 = 20$, $x - y = 2$ et $x^2 - xy + y^2 = 28$, **Find the value of** $x^3 + y^3$

Solution

$x^2 - y^2 = 20$ $\therefore (x - y)(x + y) = 20$

$x - y = 2$ $\therefore 2(x + y) = 20$ $\therefore x + y = 10$

$$\begin{aligned}x^3 + y^3 &= (x + y)(x^2 - x\,y + y^2)\\ &= 10 \times 28 = 280\end{aligned}$$

Exercices (1 - 4)

1 **Complete** to get a true sentence

A $\sqrt[3]{8x^3} = \dots\dots\dots$

B $\sqrt[3]{-125\,a^6} = \dots\dots\dots$

C $27\,m^3 = (\dots\dots\dots)^3$

D $343\,x^3\,y^6 = (\dots\dots\dots)^3$

E $x^3 - 1 = (x - 1)\,(\dots\dots\dots)$

F $8a^3 + 125 = (\dots + \dots)\,(4a^2 - 10a + \dots)$

2 **Factorize each of the following expressions:**

A $x^3 + 8$

B $m^3 + 64\,n^3$

C $x^3 - 729$

D $8 - 1000\,b^6$

E $x^{12} + y^{15}$

F $\frac{1}{8}\,a^3 - 8\,b^3$

3 **Factorize each of the following expressions:**

A $512\,x^3 - y^3$

B $343 + 27\,m^3$

C $16\,a^3\,b + 686\,b^4$

D $5\,x^3 - 40x$

E $(x + 5)^3 - 125$

F $(m - 2\,n)^3 - 8\,n^3$

4 **Factorize each of the following expressions:**

A $(x + 5)^3 + (x - 5)^3$

B $(x + y)^3 - (x - y)^3$

C $(m - n) + (m - n)^4$

D $x^6 - 7x^3 - 8$

E $0{,}027\,m^3 - n^3$

F $a^6 - 625\,b^6$

5 Let $x^3 - y^3 = 28$ and $x - y = 2$, Find the value of the expression $x^2 + x\,y + y^2$

Unit One

Lesson Five

Factorizing by Grouping

Think and Discuss

You will learn

- To factorize by grouping.

Key-Terms

- To factorize by grouping.

To factorize an algebraic expression made up of more than three terms like:

$$2ax + ay + 2bx + by$$

We notice that there is no H.C.F and it doesn't have any of the previous forms that, we have learned before. Therefore, we try to make groups having an H.C.F.

The expression $= 2ax + ay + 2bx + by$ divide into two groups

$= a(2x + y) + b(2x + y)$ H.C.F of each group

$= (2x + y)(a + b)$, $(2x+y)$ is an H.C.F of the two groups

Note that:

There is another way to regroup the expression:

The expression $= 2ax + 2bx + ay + by$ commutative property

$= 2x(a + b) + y(a + b)$

$= (a + b)(2x + y)$

Exemple

Factorize each of the following expressions:

A $x^3 + 2x^2 - x - 2$

B $16x^2 - a^2 + 6ab - 9b^2$

C $1 - x^2 - 4xy - 4y^2$

Solution

A The expression $= x^3 + 2x^2 + (-x - 2)$

$= x^2(x + 2) - (x + 2)$

$= (x + 2)(x^2 - 1)$

$= (x + 2)(x + 1)(x - 1)$

B We notice that there is no relation between the first term and all other terms. Therefore, it can be grouped this way:

$= 16x^2 - (a^2 - 6ab + 9b^2)$

$= 16x^2 - (a - 3b)^2$

$= [4x + (a - 3b)][4x - (a - 3b)] = (4x + a - 3b)(4x - a + 3b)$

C The expression $= (1) - (x^2 + 4xy + 4y^2)$

$= 1 - (x + 2y)^2$

$= (1 - x - 2y)(1 + x + 2y)$

Exercices (1 - 5)

1 **Factorize** each of the following expressions:

A $ax + bx + ay + by$

B $5L - 10m - aL + 2am$

C $am - an + m - n$

D $a^3 + a^2 + a + 1$

E $xy + 5y + 7x + 35$

F $x^3 - 3x^2 + 6x - 18$

2 **Factorize** each of the following expressions:

A $3xy - 5zL + 5zx - 3yL$

B $8mn - 2m^2 + 12nL - 3mL$

C $ab + 6mn - 2bm - 3an$

D $3ax - a - 6bx + 2b$

E $2x^2y - xy^2 + 2ax - ay$

F $abx^2 + bx - ax - 1$

3 **Factorize** each of the following expressions:

A $x^2 - 2xz - 2xy + 4yz$

B $2a^2 + 2ab + b^2 + ab$

C $a^2 + 6ab + 9b^2 - m^2$

D $9x^2 - 4a^2 + y^2 + 6xy$

E $121x^4 - 100x^2 - 20x - 1$

F $4m^4 - 9m^2 + 6m - 1$

Unit One

Lesson

Six

Factorizing by completing the square

Think and Discuss

You will learn

- To factorize by completing the square.

Key-Terms

Completing the square.

You have learned before:

A perfect square has the form $a^2 \pm 2ab + b^2$ and can be factorized in the form $(a \pm b)^2$.

There are many algebraic expression which are not in the form of a perfect square, but can be completed to have the form of a perfect square.

Exemple 1

Factorize the expression: $x^4 + 4y^4$

Solution

This expression can not be factorized according to what you have learned previously.

To factorize it, the term $2 \times \sqrt{x^4} \times \sqrt{4y^4} = 4x^2y^2$ is needed to have the form of a perfect square.

$$\begin{aligned} \textbf{Thus} &= x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2 \\ &= (x^2 + 2y^2)^2 - 4x^2y^2 \\ &= [(x^2 + 2y^2) - 2xy]\,[(x^2 + 2y^2) + 2xy] \\ &= (x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2) \end{aligned}$$

Exemple 2

Factorize the expression: $9a^4 - 13a^2b^2 + 4b^4$

Solution

The expression = $(3a^2)^2 - 13a^2b^2 + (2b^2)^2$ To be, in the form of a perfect square it should be as follows:

The middle term should be = $\pm 2 \times 3a^2 \times 2b^2 = \pm 12a^2b^2$

The expression $= (3a^2)^2 - 12\,a^2b^2 + (2b^2)^2 - a^2b^2$

$= (3a^2 - 2b^2)^2 - a^2b^2$

$= (3a^2 - 2b^2 - ab)(3\,a^2 - 2b^2 + ab)$

$= (3a^2 - ab - 2\,b^2)(3\,a^2 + ab - 2\,b^2)$

$= (3a + 2b)\,(a - b)(3a - 2\,b)\,(a + b)$

Autre solution

The expression $9a^4 - 13a^2b^2 + 4b^4$ can be factorized as a trinomial.

The expression $= (9a^2 - 4b^2)\,(a^2 - b^2)$

$= (3a + 2b)\,(3a - 2b)\,(a + b)\,(a - b)$

Using the commutative property, you get the same answer.

Exercices (1-6)

1 **Factorize each of the following expressions:**

- A $4x^4 + y^4$
- B $64\,m^4 + n^4$
- C $4x^4 + 625\,y^4$
- D $81\,x^4 + 4\,z^4$
- E $a^4 + 2500\,b^4$
- F $8x^4y^2 + 162\,z^4y^2$

2 **Factorize each of the following expressions:**

- A $x^4 + x^2\,y^2 + 25\,y^4$
- B $a^4 + 4\,a^2\,b^2 + 16\,b^4$
- C $m^4 - 11\,m^2\,n^2 + n^4$
- D $x^4 + 9x^2 + 81$
- E $16x^4 - 28\,x^2\,y^2 + 9\,y^4$
- F $4x^4 + 25\,y^4 - 29\,x^2\,y^2$

3 **Factorize each of the following expressions:**

- A $4x^2\,(4x^2 - 7y^2) + y^4$
- B $x^2\,(x^2 - 19y^2) + 25\,y^4$
- C $3m^4 + 3\,n^4 - 54\,m^2\,n^2$
- D $4a^2\,(a^2 - 6\,b^2) + 9\,b^4$
- E $9\,x^4 - 25\,x^2 + 16$
- F $x^8 - 16\,y^8$

Unit One

Lesson Seven

Solving Quadratic Equations in one Variable

Think and Discuss

You will learn

- To solve a quadratic equation in one variable.

Key-Terms

- a quadratic equation in one variable .
- Real roots of a quadratic equation.
- Solution of a quadratic equation.

You have learned before:

For any real numbers a and b, if $ab = 0$,

then $a = 0$ or $b = 0$.

Exemple If $(x - 5)(x + 2) = 0$ (1)

then: $x - 5 = 0$ or $x + 2 = 0$

$\therefore x = 5$ or $x = -2$

Note that:

❶ Each of 5 and -2 is called a root of the equation (1)

❷ The solution set is {5, -2}

Exemple 1

Find in R, the solution set of $2x^2 - 5x - 3 = 0$

Solution

By factorizing the left hand side, the equation will be in the following form.

$$(2x + 1)(x - 3) = 0$$

$$2x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$2x = -1 \quad \text{or} \quad x - 3 = 0$$

$$x = \frac{-1}{2} \quad \text{or} \quad x = 3$$

$\therefore$ **The solution set** $\{\frac{-1}{2}, 3\}$

Note that:

You can check your answer by substituting the value of x in the given equation:

For x = $\frac{-1}{2}$, **L.H.S** $= 2\left(\frac{-1}{2}\right)^2 - 5\left(\frac{-1}{2}\right) - 3$

$= 2 \times \frac{1}{4} + \frac{5}{2} - 3 = 3 - 3 = 0 =$ R.H.S.

For x = 3 **L.H.S** $= 2(3)^2 - 5(3) - 3$

$= 18 - 15 - 3 = 0 =$ R.H.S.

$\therefore$ **This means each of** $\frac{-1}{2}$, and 3 **verify the equation**

Exemple 2

Find in R the solution set of $2x^3 = 18x$

Solution

Rewrite the equation in the form

$2x^3 - 18x = 0$, then factorize.

$2x(x^2 - 9) = 0$ or $2x(x - 3)(x + 3) = 0$

$\therefore 2x = 0$ **or** $x - 3 = 0$ **or** $x + 3 = 0$

$\therefore x = 0$ **or** $x = 3$ **or** $x = -3$

$\therefore$ **S. S. = {0, 3, -3}, check your answer.**

Exemple 3

Find the real number whose double is increased by 1 than its multiplicative inverse.

Solution

Let the number be = x $(x \neq 0)$

The double of the number $= 2x$

The multiplicative inverse $= \frac{1}{x}$

$\because$ The double of the number is increased by 1 than its multiplicative inverse

$\therefore 2x - \frac{1}{x} = 1$

Multiply the both sides of the equation by x

$2x^2 - 1 = x$

$2x^2 - x - 1 = 0$

$$(2x + 1)(x - 1) = 0$$

$2x + 1 = 0$ or $x - 1 = 0$

$2x = -1$

$x = \frac{-1}{2}$ or $x = 1$

Verification:

The double of the number = -1

The multiplicative inverse = -2

Verification:

The double of the number = 2

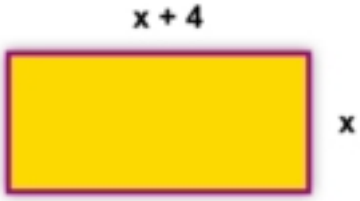
The multiplicative inverse = 1

In both cases, it is clear that the double of the number is 1 more than the multiplicative inverse.

Exemple 4

Find the dimensions of a rectangle whose length is 4cm more than its width and whose area is 21cm².

Solution



Let the width of the rectangle = x cm

$\therefore$ The length of the rectangle = (x + 4) cm

$\therefore x(x + 4) = 21$

$\therefore x^2 + 4x - 21 = 0$

$\therefore (x - 3)(x + 7) = 0$

$x - 3 = 0$ or $x + 7 = 0$

$x = 3$ or $x = -7$ (refused because it is a negative number)

$\therefore$ The width of the rectangle = 3cm, and the length of the rectangle = 3 + 4 = 7cm

Verification: area of the rectangle = $3 \times 7 = 21\text{cm}^2$

Exercices (1 - 7)

1 **Find** the solution set of the following equations in R:

A $x^2 - 8x + 15 = 0$

B $x^2 - 7x - 30 = 0$

C $6x^2 - 7x - 3 = 0$

D $5x^2 + 12x = 44$

E $(x - 3)(x + 1) = 5$

F $(x + 3)^2 - 49 = 0$

2 **Find** the solution set of the following equations in R:

A $12x^2 = 47x - 45$

B $x^4 - 5x^2 + 4 = 0$

C $(x + 3)^2 + 3(x + 3) - 10 = 0$

D $x^2 - 6x = 0$

E $4x^3 = 9x$

F $6x^2 - x = 22$

3 Find two real numbers whose product is 45 and one of them is 4 more than the other and there product is 45.

4 The length of a rectangular piece of land is more than its width by 5 meters and if its area 500m square then find its dimensions.

5 In the triangle ABC,
$m(\angle A) = (x^2 + 61)^\circ$, $m(\angle B) = (110 - 11x)^\circ$ $m(\angle c) = (90 - 7x)^\circ$.
Find the value of x and the measure of all angles.

6 Hatem is 4 years older than Hanan now, and the sum of squares of their ages now is 26. Find their ages now?

7 A right angled triangle its sides length are 2x, 2x + 1 and x - 11cm. Find the value of x and calculate the perimeter and the area of the triangle.

8 Which real number exceeds its multiplicative inverse by $\frac{5}{6}$?

9 What is the real number if added to its square results 12?

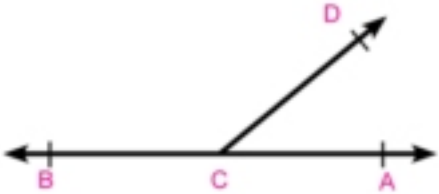
10 The sum of the squares of two succesive odd numbers is 130. Find the two numbers.

General Exercises

1. **Factorize each of the following:**
   - A $x^4 - 16y^4$
   - B $2x^5 + 54x^2$
   - C $a^4 + 4b^4$
   - D $x^6 - 64y^6$
   - E $8x^3 - 125$
   - F $3x^3 + 2x^2 + 12x + 8$
2. **Factorize each of the following:**
   - A $8x^2 - 2xy - y^2$
   - B $L^3m - 27m^4$
   - C $625a^2 - 81b^2$
   - D $2(x + 3y)^3 - 250$
   - E $(c - d) + 2x(c - d) + x^2(c - d)$
   - F $7x^2 - 29xy + 30y^2$
3. **Find the value of c, where $c \in z$, so that the given expression can be factorized and then factorize it:**
   - A $x^2 + cx - 15$
   - B $x^2 - 7x + c$
   - C $y^2 - cy + 29$
   - D $a^2 + a - c$
   - E $cx^2 + x - 15$
   - F $cx^2 - 13x + 6$
4. **Factorize each of the following expressions:**
   - A $9x^2 - 30x + 25$
   - B $18ab^4 - 114b^2c^2a + 128ac^2$
   - C $x^2 - 4xy + x - 2y + 4y^2$
   - D $x^2 - 2xy + y^2 - 4z^2$
5. **Find the solution set of each of the following equations:**
   - A $x^2 + x = 6$
   - B $3x^2 + 2x = 85$
   - C $(x - 1)^2 + x = 3$
   - D $2x^3 = 7x$
6. The sum of three successive integers is equal to the square of its middle integer. Find these numbers.
7. In the opposite figure: $\overrightarrow{CD} \cap \overleftrightarrow{AB} = \{C\}$

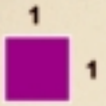
   $m(\angle BCD) = (x^2)^\circ$ and $m(\angle ACD) = (8x)^\circ$,

   Find x.

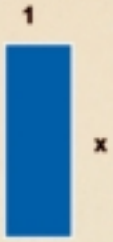


Activité

You can use the following units to explain some factorization problems:

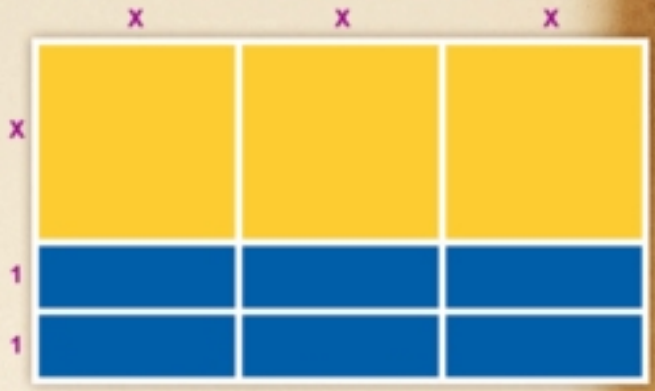


The area of the square = 1



The area of the rectangle = x

The area of the square = x^2



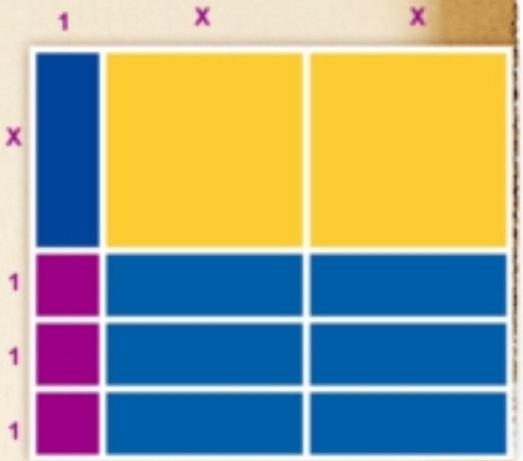
1 **The area of the opposite figure is $3x^2 + 6x$**

Note that:

The dimensions of the rectangle are 3x and x+2

Its area $= 3x(x + 2)$

$= 3x^2 + 6x$



2 The area of the opposite figure = ...

The dimensions of the rectangle are ......and ...

What is the relation between the area and the dimensions of the rectangle?

3 Sketch the figure that represents the area:

$3x^2 + 7x + 2$

Unit Test

1 **Choose the correct answer:**

A The quadratic trinomial expression $4x^2 + k + 25y^2$ is a perfect square when k = ... (20 or $10\,x\,y$ or $\pm 20\,x\,y$ or $30\,x\,y$)

B If $x^2 - y^2 = 16$ and $x + y = 8$, then $x - y =$ ....(2, 1, 128, 64)

C If $x + y = 3$ and $x^2 - x\,y + y^2 = 5$, then $x^3 + y^3 =$ ....(15, 25, 8, 7)

D The quadratic trinomial expression $4\,x^2 + 12x + a$ is a perfect-square when = ...(6,16,1, 9)

E If $(2a - 5)\,(3a - 2) = 6a^2 + ka + 10$, then k = ..($15$, 19, -19, 4)

2 **Find the missing term to get a true sentence:**

A $(4a - 5b)$ (....... $- 3\,b$) $= 8a^2$ .... $+ 15\,b^2$

B If $x^2 + y^2 = 17$ and $x\,y = 7$, then $(x - y)^2 =$ ....

C If $k\,x^2 - 10x + 1$, is a perfect-square, then k = ...

D If $(x + 1)$ is one of the linear factors of the expression $5x^2 - 2x - 7$ then the other factor = ...

E $x^3 + 8 = (\,x + 2)$ (......................)

3 **Factorize each of the following expressions:**

A $(x + 2)^3 - 4\,x - 8$

B $a^2 + 2a\,b + b^2 - C^2$

C $2\,x^2 - 5x + 3$

D $x^4 + 4\,L^4$

E $8x^3 - 343\,y^6$

4 **Solve each of the following equations in R:**

A $x^2 - 3x - 10 = 0$

B $3\,x^2 + x = 14$

C $(2x-1)^2 + (x-1)^2 = 10$

5 **Evaluate using the factorization:**

A $\times 75 - \frac{20}{7} \times 5$

B $(8.175)^2 - (1.825)^2$

C $(87)^2 + 2 \times 13 \times 87 + (13)^2$

6 **In a right angled triangle the length of the two sides of the right angle are 4x and x + 1 cm. Find the length of the hypotenuse if the area of the triangle is 84cm².**

Algebra

Unit 1: Non-Negative and negative Integer Powers in R and the Operations on them

lesson 1 : Non- Negative and negative integer powers in R.

lesson 2 : Rules of non-negative integer powers in R.

lesson 3 : Rules for negative integer powers in R.

lesson 4 : Operations on integer powers.

Unit TWO

Lesson One

Non - Negative and negative Integer Powers in R

Think and Discuss

What you'll learn

- ☆ Non-negative and negative intger powers

Key terms

- ☆ R^* the set of real numbers except zero
- ☆ Non-negative integer powers in R
- ☆ Negative intger powers in R
- ☆ Exponential equation in R

First: Non-negative intger powers:

You have previously learned the integer powers in the set of rational numbers Q :

Complete :

① $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = (\ldots\ldots\ldots)^{\ldots}$ ② $\frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = (\ldots\ldots\ldots)^{\ldots}$

if $a \in R, n \in Z^+$ then $a^n = a \times a \times a \times \ldots\ldots\ldots \times a$

where a is repreated as a factor n times

Example

① $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} = (\sqrt{2})^5 = 4\sqrt{2}$.

② $-\sqrt{2} \times -\sqrt{2} \times -\sqrt{2} \times -\sqrt{2} = (-\sqrt{2})^4 = 4$

③ $-\sqrt{5} \times -\sqrt{5} \times -\sqrt{5} = (-\sqrt{5})^3 = -5\sqrt{5}$

If $a \in R^*$ then $a^{zero} = 1$

for example: $(\sqrt{7})^{zero} = 1$, $(\frac{-1}{\sqrt{11}})^{zero} = 1$

Second: Negative intgerr powers

Think and Discuss

You know that $5^3 \times 5^{-3} = 5^{3-3} = 5^0 = 1$

Complete:

$x^m \times \ldots\ldots\ldots = 1$ **where** $x \neq 0$, $m \neq 0$

If $a \in R^*$, $n \in Z^+$

then $a^{-n} = \frac{1}{a^n}$, $a^n = \frac{1}{a^{-n}}$

for example: $(\sqrt{3})^{-4} = \frac{1}{(\sqrt{3})^4} = \frac{1}{9}$, $\frac{1}{(-\sqrt{3})^{-3}} = (-\sqrt{3})^3 = -3\sqrt{3}$

Drill

If $x = 3$, $y = \sqrt{2}$, then find each of the following in the simplest form:

① $x^{-2} y^{-4}$ ② $(x^{-2} \times y^4)^{-2}$ ③ $\left(\frac{x}{y}\right)^{-3}$

Example

① If $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}}$ and $z = \frac{\sqrt{2}}{2}$. **then find the value of:** $x^2 + (x\,z)^2 \times y^2$

Solution

The expression $= x^2 + x^2 z^2 y^2 = x^2 (1 + z^2 y^2)$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 \times [1 + \left(\frac{\sqrt{2}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2] = \frac{3}{4} \times [1 + \frac{2}{4} \times \frac{1}{3}] = \frac{3}{4} \times \frac{7}{6} = \frac{7}{8}$$

Important Rule :

If $a^m = a^n$ **then $m = n$ *where* $a \in R - \{0, 1, -1\}$**

For example: If $(\sqrt{2})^x = 2\sqrt{2}$ then $(\sqrt{2})^x = (\sqrt{2})^3$ $\therefore x = 3$

If $a^n = b^n$ **then $a = b$ *where* $n \in \{1, 3, 5, ...\}$**
, $|a| = |b|$ *where* $n \in \{2, 4, 6, ...\}$

For example: $x^5 = \frac{1}{32}$ then $x^5 = (\frac{1}{2})^5$ $\therefore x = \frac{1}{2}$

② **Find the solution set for each of the following equations in R :**

A $(\frac{3}{5})^{x+2} = \frac{125}{27}$ B $(3)^{x-3} = (\sqrt{3})^{x+5}$

Solution

A $(\frac{3}{5})^{x+2} = \frac{125}{27}$ $\therefore (\frac{3}{5})^{x+2} = (\frac{5}{3})^{3}$ $\therefore (\frac{3}{5})^{x+2} = (\frac{3}{5})^{-3}$

$\therefore x + 2 = -3$ $\therefore x = -2 - 3$ $\therefore x = -5$

The Solution Set is {-5}

B $\because [(\sqrt{3})^{2}]^{(x-3)} = (\sqrt{3})^{(x+5)}$ $\therefore (\sqrt{3})^{2x-6} = (\sqrt{3})^{x+5}$

$\therefore 2x - 6 = x + 5$ $\therefore x = 11$ The Solution Set is {11}

Drill *Mental math*

Solve by inspection $\frac{1}{(x+9)^{4}} = 0.0001$ What do you notice?

Exercises (2-1)

First **Choose the correct answer :**

1. $4^{3} + 4^{3} + 4^{3} + 4^{3}$ equals: A 4^{3} B 4^{4} C 4^{12} D 4^{81}
2. 0.002×0.05 equals: A 10^{-5} B 10^{-4} C 10^{4} D 10^{5}
3. If $x = \frac{\sqrt{9}}{\sqrt{3}}$ then x^{-1} equals: A $\frac{\sqrt{3}}{3}$ B $\frac{\sqrt{3}}{\sqrt{2}}$ C $\sqrt{3}$ D 2
4. If $5^{x} = 4$ then 5^{x-1} equals: A 1.25 B 0.8 C 0.125 D 0.08
5. If $(x - 5)^{zero} = 1$ then $x \in$ ………… A $R - \{5\}$ B $R - \{-5]$ C $\{5\}$ D R

Second **Find the value of the following in the simplest form:**

1. 3^{-1}
2. $(\frac{1}{4})^{-1}$
3. $(\frac{3}{2})^{-3}$
4. $(\sqrt{5})^{4}$
5. $(-\sqrt{3})^{-2}$
6. $(\sqrt[3]{7})^{-3}$
7. $(\frac{-1}{\sqrt{2}})^{6}$
8. $(0.01)^{-2}$
9. $(\frac{-\sqrt{2}}{2})^{-4}$

Third

1. If $x = 2$, $y = \sqrt{3}$, **then find the value of the following in the simplest form:**

   A $3(x+y)^{4}(x-y)^{4}$ B $(\frac{x+y}{x-y})^{-2}$

2. **Find the value of x in each of the following:**

   A $3^{x-2} = 81$ B $(\sqrt{3})^{x-1} = 9$ C $(32)^{x-3} = 8^{2x+1}$

   D $25 \times 3^{x-1} = 9 \times 5^{x-1}$

Unit TWO

Lesson TWO

Rules of non - negative integer powers in R

Think and Discuss

First:

Complete: $(\sqrt{3})^2 \times (\sqrt{3})^4 = (\dots\dots\dots)^{\dots\dots}$ **What do you notice?**

If $a \in R^*$, m, n *are two non-negative integer numbers,*

then: $a^m \times a^n = a^{m+n}$

Generalization:

If $a \in R^*$, m, n, ..., ℓ are non-negative integer numbers

then: $a^m \times a^n \times \dots\dots\dots \times a^\ell = a^{m+n+\dots\dots\dots+\ell}$

From the previous rule, we find that: $(\sqrt{3})^2 \times (\sqrt{3})^4 = (\sqrt{3})^{2+4} = (\sqrt{3})^6 = 27$

Second

Complete : $(\sqrt{5})^7 \div (\sqrt{5})^3 = (\dots\dots\dots)^{\dots\dots}$ **What do you notice?**

If $a \in R^*$ *, and m, n are two non-negative integer numbers*

***$m \geqslant n$* then $a^m \div a^n = a^{m-n}$**

From the previous rule we find that: $(\sqrt{5})^7 \div (\sqrt{5})^3 = (\sqrt{5})^{7-3} = (\sqrt{5})^4 = 25$

Third :

Complete: $(\sqrt{2} \times \sqrt{3})^2 = (\sqrt{2})^{\dots\dots} \times (\dots\dots)^{\dots\dots} = \dots\dots \times \dots\dots = \dots\dots$

If a and b $\in R^*$, n *is a non-negative integer numbers*

then: $(ab)^n = a^n \times b^n$

Generalization:

If a, b, c, ..., $k \in R^*$, n is a non-negative integer number then:

$(a \times b \times c \times \dots \times k)^n = a^n \times b^n \times c^n \times \dots \times k^n$

What you'll learn

- ☆ Rules of non-negative integer in R
- ☆ Solving problems on non-negative integer powers in R.

Key terms

- ☆ Non-negative integer powers.
- ☆ Set of real numbers.

Fourth:

Complete : $\left(\frac{\sqrt{3}}{\sqrt{5}}\right)^4 = \frac{(\ldots\ldots)^{\ldots\ldots}}{(\ldots\ldots)^{\ldots\ldots}} = \frac{\ldots\ldots}{\ldots\ldots}$

If $a, b \in R^*$, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, *n is a non-negative integer*

where $b \neq 0$, $a \neq 0$

Generalization : If a, b, c, ...   , $k \in R$ and n is a non-negative integer then:

$\left(\frac{a\,b\,e \times \ldots\ldots \times \ell}{c\,d\,f \times \ldots\ldots \times k}\right)^n = \frac{a^n b^n e^n \ldots\ldots \ell^n}{c^n d^n f^n \ldots\ldots k^n}$ where any of the factors of the denominator ≠ zero

Fifth:

Complete : $(2^2)^3 = (\ldots\ldots)^{\ldots\ldots} \times (\ldots\ldots)^{\ldots\ldots} \times (\ldots\ldots)^{\ldots\ldots} = (\ldots\ldots)^{\ldots\ldots}$.**What do you notice?**

If $a, b \in R^*$, m, n are *two non-negative integers* then $(a^m)^n = a^{mn}$

Generalization: If a, b and c, ..., $k \in R$ and n is a non-negative integer, then :

$\left(\frac{b^m e^\ell \ldots\ldots}{f^k d^x \ldots\ldots}\right)^n = \frac{b^{nm} e^{n\ell} \ldots\ldots}{f^{nk} d^{nx} \ldots\ldots}$ Where any of the factors of the denominator ≠ 0

Example

Simplify each of the following to the simplest form:

① $\sqrt{2} \times (\sqrt{2})^2 \times (\sqrt{2})^3$ ② $\left((\sqrt{2})^3 \times (-\sqrt{2})^2\right)^2$ ③ $\frac{(\sqrt{3})^5 \times (\sqrt{3})^3}{(\sqrt{3})^4}$

Solution

① $\sqrt{2} \times (\sqrt{2})^2 \times (\sqrt{2})^3 = (\sqrt{2})^{1+2+3} = (\sqrt{2})^6 = 8$

② $\left((\sqrt{2})^3 \times (-\sqrt{2})^2\right)^2 = (\sqrt{2})^{3\times 2} \times (-\sqrt{2})^{2\times 2} = (\sqrt{2})^6 \times (-\sqrt{2})^4 = 8 \times 4 = 32$

③ $\frac{(\sqrt{3})^5 \times (\sqrt{3})^3}{(\sqrt{3})^4} = (\sqrt{3})^{5+3-4} = (\sqrt{3})^4 = 9$

Exercises (2-2)

First **Choose the correct answer :**

1. Which of the following is the nearest to $11^2 + 9^2$?

   A $22 + 18$ B $211 + 29$ C $120 + 20$ D $120 + 80$

2. The value of: $(2)^{20} + (2)^{21}$ is:

   A 2×2^{40} B 2×2^{41} C 3×2^{20} D 3×2^{21}

3. The value of: $(3)^{\text{zero}} + (-\frac{1}{\sqrt{3}})^2 + \frac{1}{\sqrt[3]{-27}}$ is :

   A zero B $\frac{1}{3}$ C 1 D 3

4. One sixth of: $2^{12} \times 3^{12}$ is:

   A 6^2 B 6^4 C 6^{11} D 6^{23}

5. The value of : $2^5 + (\sqrt{2})^{10}$ is :

   A 2^6 B 2^{10} C $(\sqrt{2})^{15}$ D $(\sqrt{2})^{20}$

Second **Find the simplest form :**

1. $(\sqrt{2})^2 \times (\sqrt{2})^4$
2. $(-\sqrt{5})^9 \div (-\sqrt{5})^5$
3. $(\frac{3\sqrt{2}}{2\sqrt{3}})^4$
4. $\frac{(\sqrt{3})^7 \times (\sqrt{3})^8}{(\sqrt{3})^6}$

Third

1. If $a = \frac{1}{\sqrt{2}}$, $b = -1$ then calculate the value of $7a^6 + (1 - b)^{-3}$

2. If $a = \sqrt{3}$, $b = \sqrt{2}$ then calculate the value of:

   A $a^4 - b^4$. B $\frac{a^4}{b^4}$.

3. If $x = 2\sqrt{2}$, $y = 3$ then calculate the value of: $(x^2 - y^2)^3$.

4. If $(\sqrt{\frac{3}{2}})^x = \frac{4}{9}$. then calculate the value of $(\frac{3}{2})^{x+1}$.

Unit TWO

Lesson Therr

Rules for negative integer powers in R

Think and Discuss

What you'll learn

- ☆ Generalization of the laws for non-negative and negative powers in R.

Key Terms

- ☆ Negative integer powers
- ☆ Set of real number R.

Generlization the laws of exponents

If a, and b $\in R^*$, m, and n $\in Z$ then:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a\,b)^n = a^n \times b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $(a^m)^n = a^{m\,n}$

Remarks:

1. If $a \in R^*$, $n \in Z^+$ then a^n, a^{-n} each is a multiplicative inverse for the other, then , $a^n \times a^{-n} = 1$ *for example:* $(\sqrt{3})^5 \times (\sqrt{3})^{-5} = 1$
2. If a, b $\in R^*$, $n \in Z^+$ then $\left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}$

Example: $\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5 = \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{-5}$,where: $\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5 \times \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{-5} = 1$

Example

1 Find in the simplest form :

A $5\,(\sqrt{5}\,)^{-1}$ B $\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{-4}$ C $\frac{2^{-1} \times 4}{3^{-1}}$

Solution

A $5\,(\sqrt{5}\,)^{-1} = \frac{5}{\sqrt{5}} = \frac{5}{5\sqrt{\ }} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

B $\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{-4} = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^{+4} = \frac{4}{9}$

C $\frac{2^{-1} \times 4}{3^{-1}} = \frac{3 \times 4}{2} = 6$

② Find in the simplest form $\dfrac{(15)^{-2} \times (\sqrt{5})^{3} \times (3)^{3}}{9 \times (\sqrt{5})^{-3}}$

Solution

Expression $= \dfrac{(3)^{-2} \times (5)^{-2} \times (\sqrt{5})^{3} \times (3)^{3}}{(3)^{2} \times (\sqrt{5})^{-3}} = (3)^{-2+3-2} \times (5)^{-2} \times (\sqrt{5})^{3+3}$

$= (3)^{-1} \times (5)^{-2} \times (\sqrt{5})^{6} = \dfrac{1}{3} \times (5)^{-2} \times (5)^{3} = \dfrac{1}{3} \times (5)^{1} = \dfrac{5}{3}$

③ If $\dfrac{49^{n} \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343,$ Then calculate the value of 6^{2n}

Solution

$\because \dfrac{49^{n} \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$ $\therefore \dfrac{7^{2n} \times 5^{4n} \times 3^{4n}}{7^{-n} \times 5^{4n} \times 3^{4n}} = 343$

$\therefore 7^{2n+n} = 343$ $\therefore 7^{3n} = 7^{3}$

$\therefore 3n = 3$ $\therefore n = 1$ $\therefore 6^{2n} = 6^{2\times1} = 36$

Exercises (2-3)

First Complete :

① The simplest form of the expression: $2^{\text{zero}} + (2)^{-1} \left(-\dfrac{1}{\sqrt{2}}\right)^{2} =$ ....

② If $x = (\sqrt{2} + 3)^{5}$, $y = (\sqrt{2} + 3)^{-5}$ then $x\,y =$ ....

③ $a^{-4} + 1 = a^{-4}$ (.......... + ...) where $a \neq 0$

④ If $2^{x} \times 5^{-x} = 2.5$ then $x =$ ....

⑤ If $4^{x-10} = \frac{1}{16}$ then $\sqrt[3]{x} =$ ....

Second Choose the correct answer:

① If $(x-3)^{\text{Zero}} = 1$ then $X \in$ ....
Ⓐ $R - \{3\}$ Ⓑ $R - \{-3\}$ Ⓒ $\{3\}$ Ⓓ R

② $(\sqrt{3} + \sqrt{2})^{9} (\sqrt{3} - \sqrt{2})^{9}$ is:
Ⓐ 1 Ⓑ $\sqrt{5}$ Ⓒ $\sqrt{6}$ Ⓓ 5

③ If $3^{x} = 5$ et $\dfrac{1}{3^{y}} =$ then $3^{x+y} =$ ....
Ⓐ $\frac{5}{7}$ Ⓑ $\frac{7}{5}$ Ⓒ 2 Ⓓ 12

④ If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$ then $x =$ ....
Ⓐ -3 Ⓑ -1 Ⓒ 1 Ⓓ 3

Third Find in the simplest form in each of the following :

① $\left(\frac{\sqrt{3}}{3}\right)^{-5}$

② $(\sqrt{3})^{-4} \times (-\sqrt{2})^{4}$

③ $\left(\frac{1}{\sqrt{3}}\right)^{5} \div \left(\frac{1}{\sqrt{3}}\right)$

Fourth Find in the simplest form in each of the following :

① $\frac{(\sqrt{3})^{-5} \times (\sqrt{3})^{-4}}{(\sqrt{3})^{-10}}$

② $\frac{(10)^{2} \times (10)^{-7}}{(0.1)^{2} \times 0.001}$

Fifth Find the value of x in each of the following :

① $2^{x} = 32$

② $2^{x-3} = 1$

③ $3^{x-2} = \frac{1}{9}$

④ $\left(\frac{2}{5}\right)^{2x-1} = \frac{8}{125}$

⑤ $\left(\frac{2}{3}\right)^{x-4} = 2\frac{1}{4}$

Sixth

① If $3^{x} = 27$ and $4^{x+y} = 1$. Calculate the value of x and y.

② If: $\frac{8^{x} \times 9^{x}}{(18)^{x}} = 64$ find x, then Calculate the value of $(4)^{-x}$

③ Find in the simplest form of $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$ then calculate the result when x = 1.

Unit TWO
Lesson
Four

Operations on Integer Powers

Think and Discuss

First: Find each of the following in the simplest form:

① $\frac{1}{(\sqrt{3})^5} \div 9\sqrt{3}$ $\qquad$ $\frac{3\sqrt{2}}{\sqrt{3}} - \frac{(\sqrt{3})^3}{2\sqrt{2}}$

We have previously learned that :

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ *(where* a *and* b, d ≠ 0*)*

$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ *(where* b *and* c, d ≠ 0*)*

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ *(where* b, d ≠ 0*)*

$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$ *(where* b, d ≠ 0*)*

Second: Use mental math to find: $3 \times 2^2 - 6 \div 3 \times 5 + 4$.

Check your answer using the calculator to do the operation above.

Ordering operations :

① Do the operations in the interior parenthesis, then the exterior parenthesis if found.

② Calculate the powers of numbers.

③ Do multiplication or division from left to right.

④ Do addition or subtraction from left to right.

This order is followed in calculators.

What you'll learn

- ☆ Do operation (+ , - , ×, ÷) on integer powers.

Key terms

- ☆ Non-negative integer powers.
- ☆ Negative integer powers Ordering operation.

Example

① **Find the result of each of the following in the simplest form:**

A $2^{-3} \times 3^{-2} \div 6^{-4}$

B $(\sqrt{5})^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3}$

Solution

A $2^{-3} \times 3^{-2} \div 6^{-4} = 2^{-3} \times 3^{-2} \times 6^{4}$

$= 2^{-3} \times 3^{-2} \times 2^{4} \times 3^{4} = 2^{-3+4} \times 3^{-2+4}$

$= 2^{1} \times 3^{2} = 2 \times 9 = 18$

Calculators are used to check the previous operations as follows :

Start $\rightarrow$ $2\ \boxed{x^{\blacksquare}}\ \boxed{(-)}\ 3\ \boxed{\times}\ 3 \blacktriangleright \boxed{x^{\blacksquare}}\ \boxed{(-)}\ 2 \blacktriangleright \boxed{\div}\ 6\ \boxed{x^{\blacksquare}}\ \boxed{(-)}\ 4 \blacktriangleright \boxed{=}$

B $(\sqrt{5})^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3} = (\sqrt{5})^5 \div (\sqrt{5})^3 + 2(\sqrt{3})^2$

$= (\sqrt{5})^{5-3} + 2 \times 3 = (\sqrt{5})^2 + 6 = 5 + 6 = 11$

② If $\frac{3^x \times 8^x}{12^{x+1}} = \frac{1}{3}$ **Find the value of x**

Solution

$$\frac{3^x \times 2^{3x}}{(2^2 \times 3)^{x+1}} = \frac{1}{3}$$

$$\frac{3^x \times 2^{3x}}{3^{x+1} \times 2^{2x+2}} = \frac{1}{3}$$

$$3^{x-x-1} \times 2^{3x-2x-2} = \frac{1}{3}$$

$$3^{-1} \times 2^{x-2} = \frac{1}{3}$$

$$\frac{1}{3} \times 2^{x-2} = \frac{1}{3}$$

$$2^{x-2} = 1$$

$$2^{x-2} = 2^0 \rightarrow x - 2 = 0 \rightarrow x = 2$$

③ **If $a = \sqrt{2}$, $b = \sqrt{3}$. Find the numerical value of:**

A $\dfrac{b^4 - a^4}{b^2 + a^2}$ **B** $\dfrac{a^3 + b^3}{a + b}$

Solution

A $\dfrac{b^4 - a^4}{b^2 + a^2} = \dfrac{(b^2 + a^2)(b^2 - a^2)}{b^2 + a^2}$

$= b^2 - a^2 = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$

B $\dfrac{a^3 + b^3}{a + b} = \dfrac{(a + b)(a^2 - ab + b^2)}{a + b} = a^2 - ab + b^2$ $(a \neq -b)$

$= (\sqrt{2})^2 - \sqrt{2} \times \sqrt{3} + (\sqrt{3})^2 = 2 - \sqrt{6} + 3 = 5 - \sqrt{6}$

Drill

① If $\dfrac{6^{2n} \times 2^{2n}}{4^{2n} \times 3^{2n+4}} = 9^{-x}$ **Find the value of x**

② **Connecting with commercial business.**

If $c = m(1 + r)^n$ where (c) is the total sum m in pounds, (r) is the profit per pound yearly and n is the number of years, then calculate (c) to the nearest pound If $m = 2.5 \times 10^4$, $r = 9.8 \times 10^{-2}$, $n = 12$.

General Exercises

First Complete

① The Simplest form of the expression : $2^{-3} \times 2^{-2} \div 4^{-3} = \dots\dots$

② The Simplest form of the expression: $(3^{-2})^3 \div 9^{-3} \times (-2)^{-1} = \dots\dots$

③ The Simplest form of the expression: $4^3 \times 3^{-2} \times (\sqrt[3]{-8})^{-5} = \dots\dots$

④ If: $3^x + 3^x + 3^x = 1$ then $x = \dots\dots$

⑤ $\frac{2^x \times 3^x}{(12)^x} = \frac{1}{2}$ then $x = \dots\dots$

Second Choose the correct answer :

① the expression : $\frac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x}$ equals

A 3^{2x-1} B 3^{1-2x} C 3^{x^3-3x} D 3^{3x-x^3}

② The numerical value of the expression $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$ is :

A $\frac{1}{10}$ B 7 C 10 D 100

③ $(5^{x+2} - 5^{x+1}) \div 5^x =$

A 5 B 10 C 15 D 20

④ The value of the expression $3^5 + (\sqrt{3})^{10} - 2(3)^5 = \dots\dots$

A Zero B 3^5 C $(\sqrt{3})^5$ D $2(3)^5$

⑤ $6^x = 11$ then $6^{X+1} = \dots\dots$

A 12 B 22 C 66 D 72

Third

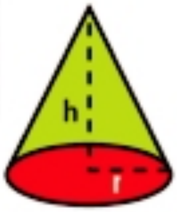
① If $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$, then find the value of the expression: $\frac{x^7 y^8 - y}{(x+y)^9}$ in the simplest form .

② If $\frac{8^x \times 9^x}{18^x} = 64$ Find X then find the value of 4^{-x}

③ **Simplify** $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$ **then find the value of the result at X=1**

④ (Connecting with Geometry) If the total area of a cube is 3.375×10^2 cm², unit area, then find : A Edge length of the cube B Volume of the cube

⑤ (Conneting with Geometry) If the volume of a circular right cone gives the relation $V = \frac{1}{3} \pi r^2 h$. then find the height of the cone if its volume is 7.7×10^2 cm³ and the diameter length of its base is 14 cm. $[\pi = \frac{22}{7}]$



⑥ (Connecting with Geometry) If the volume of a sphere $V = \frac{4}{3} \pi r^3$ then find the radius length of a sphere with a volume 3.8808×10^4cm³ $[\pi = \frac{22}{7}]$

⑦ If : $c = \frac{a(r^n - 1)}{r - 1}$ and a= 128, $r = \frac{3}{2}$, $c = 6.305 \times 10^3$. then find n.

Connecting with technology

To find the sum of the expression: $\frac{(15)^{-2} \times (\sqrt{5})^3 \times (3)^3}{9 \times (\sqrt{5})^{-3}}$ (The result $= \frac{5}{3}$)

Hint

Follow the following steps using the calculator:

Start → 15 $x^{■}$ (−) 2▶ × √■ 5 $x^{■}$ 3▶ × 3 $x^{■}$ 3▶

= ÷ (9 × √■ 5 $x^{■}$ (−) 3▶) =

Activites

① *Find* **in the simplest form:** $(\sqrt{3}+2)^{11}(\sqrt{3}-2)^{11}$

② *If* **$a=\sqrt{7}$, $b=(\sqrt{7})^{-1}$. then find the value of:** $a^{101}b^{100}$

③ *Find* in the simplest form:

$$\frac{1}{1-\sqrt{3}}-\frac{1}{\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{7}}-\frac{1}{\sqrt{7}-\sqrt{9}}+\frac{1}{\sqrt{9}-\sqrt{11}}-\frac{1}{\sqrt{11}-\sqrt{13}}+\frac{1}{\sqrt{13}-\sqrt{15}}$$

Multiply each fraction by the conjugate of its denominator up and down.

Unit test

First **Choose the correct answer:**

① The digit in the units place of $3^{12}\times 2^{14}$ is : Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ 6 Ⓔ 8

② If : $x\neq 0$, $x+\frac{1}{x}=\sqrt{5}$ then $x^2+\frac{1}{x^2}=$ Ⓐ 1 Ⓑ 3 Ⓒ 5 Ⓓ 7 Ⓔ 9

Second

① **Simplify :**

Ⓐ $((-5)^3)^2\times(-\sqrt{5})^{-4}$ Ⓑ $\frac{8^{n-1}\times 32^{-n}}{32\times 4^{-n}}$

② **Calculate the value of x:**

Ⓐ $(\frac{2}{3})^{x+5}=(3\frac{3}{8})^{-2}$ Ⓑ $5^{x^2-5x}=0.0016$

-n

③ If $(\sqrt{\frac{3}{2}})^x=\frac{9}{4}$ Find $(\frac{3}{2})^{x+1}$

④ **Population: If the number of population (y) in million in a country is identified by the relation** $(y)=11.7\,(1.02)^x$ where x the number of years starting from year 2005.

Calculate the number of population expected for this country to the nearest million

Ⓐ year 2011 Ⓑ year 2000

UNIT THREE

3

Probability

Unit THREE

Lesson One

Probability

Think and Discuss

You will learn

- The meaning of inferential statistics.
- The concept of a sample.
- Random experiment.
- Sample space.
- Event.
- The concept of probability.
- Prediction.

Key-Terms

- Sample.
- Random experiment.
- Sample space.
- Event.
- Probability.
- Prediction.

You have learned before some statistical ways and procedures used in collecting and organizing data, to display data in tables and graphs and to use frequency tables or grouped frequency tables (ascending and descending). You have learned also to organize data sets by using bar graphs, line graph, histograms. frequency tables......

You have learned also how to express data in brief forms by finding mean, median, and mode used to estimate and make decisions.

Statistical inference:

Let's think

A feasibility study is always needed before starting build a factory or any investment project.

The quality assurance of production of a factory shows that 2% of the production of a certain machine is defective. What is meant by this?

A feasibility study is considered predictional way about the success of the project and achieving its objectives.

So It is necessary to start first with formulating hypotheses about the location of the project, operating supplies, employment and marketing procedures, then testing and checking these hypotheses to make a decision to start the project.

2% of the production of machines is a defective production this does not mean that each produced 100 units, you will find 2 units out of order.

Therefore, the percentage 2% means the averange of defective units when examining a large number of production sets, each set consists of 100 units. There fore the probability of producing a defective unit is 0.02.

Therefore:

Statistical inference depends on the process of producing accurate statistics and requires careful planning and selecting a representative sample of the population.

Probability is used to support conclusions made from results of a survey in many samples.

Think

What are samples and types of samples? how can a random sample be chosen? How can a regular sample be chosen? Why are samples used?

The concept of a sample

A sample is any part of a population. To obtain information about a large group, or population, smaller parts or samples are studied. A sampling method is a procedure for selecting a sample to represent the popoulation and to provide a reasonable representation of a population situations.

Probabilities are used in making decisions from a set of avilable decisions concerning with studying of a certain phenomenan in case of uncertainty or encountring the imperfect data .

Probability:

You have learned before the theoritical and experimental probability. Experimental probability depends on experiments and results of a survey.

Anyway, the probability of an event is described by the ratio:

$$\text{The probability of an event} = \frac{\text{number of outcomes in the event}}{\text{number of all possible outcomes in the sample space}}$$

As the number of trials in an experiment increases, the approximation of the experimental Probability improves and becomes closer to the theoritical probability.

Therefore, The expected number of outcomes in an event = the probability of its occurrence X number of all possible outcomes.

Theoritical probability is based on the assumption that all outcomes in the sample space occur randomly, which means all possible outcomes are equally likely. For example:

❶ Tossing a regular coin. There are 2 possible ways the coin can land; heads (H) or tails (T). Each way has the same chance of happening. The chances of heads and tails are equally likely.

❷ Rolling a regular die and observing the number on the upper face. Each number has the same chance of occurring. The chances of all numbers are equally likely.

❸ Drawing a colored marble from a bag containing similar colored marbles with the same volume and the same number of each color. The chances of all outcomes are equally likely .

❹ Drawing a card from a set of similar cards and recording what is written on it ....etc.

A random experiment is an experiment, where its all possible outcomes are known before simulating it but we can't determine the actual outcome.

Sample spaces is the set of all possible outcomes of a random experiment. The number of its elements is denoted by n (s)

An event is a subset of the sample space. If A is an event in S, then $A \subset S$, and the number of elements in A is denoted by n(A) and is the number of outcomes in the event A.

Then: probability of occuring an event $A \subset S$, is denoted by P(A),

where:

$$P(A) = \frac{\text{number of outcomes in the event A}}{\text{number of all possible outcomes in the sample space}} = \frac{n(A)}{n(S)}$$

$$\because n(A) \leqslant n(S) \qquad \therefore \frac{n(A)}{n(S)} \leqslant 1$$

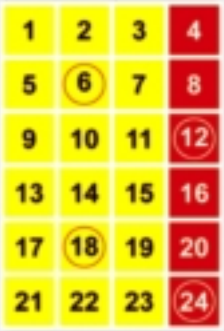
$$\because n(A) \in N,\ n(S) \in Z^{+} \qquad \therefore \frac{n(A)}{n(S)} \geqslant 0$$

$$\therefore 0 \leqslant \frac{n(A)}{n(S)} \leqslant 1 \qquad \text{i. e } 0 \leqslant p(A) \leqslant 1$$

Exemple (1) :

A numbered card is selected randomly from a set of similar cards numbered from 1 to 24 Find the probability of getting a card carries:

- **A** a multiple of 4
- **B** a multiple of 6
- **C** a multiple of 4 and 6 together.
- **D** a multiple of 4 or 6
- **E** a number divisible by 25
- **F** a positive integer less than 25



1	2	3	4
5	⑥	7	8
9	10	11	⑫
13	14	15	16
17	⑱	19	20
21	22	23	㉔

Solution

Sample Spaces = $\{1, 2, 3, ..., 24\}$
$n(S) = 24$

A **Let A be the event of getting a multiple of 4**

$\therefore A = \{4, 8, 12, 16, 20, 24\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{24} = \frac{1}{4}$$

B **Let B be the event of getting a multiple of 6**

$B = \{6, 12, 18, 24\}$, $n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{24} = \frac{1}{6}$$

C Let C be the event of getting a multiple of 4 and 6 together.

$C = \{12, 24\}$, $n(C) = 2$

$$P(C) = \frac{n(C)}{n(E)} = \frac{2}{24} = \frac{1}{12}$$

D Let D be the event of getting a multiple of 6 or a multiple of 4 ».

$D = \{4, 8, 12, 16, 20, 24, 6, 18\}$

$n(D) = 8$

$$P(D) = \frac{n(D)}{n(E)} = \frac{8}{24} = \frac{1}{3}$$

E Let E be the event of getting a number divisible by 25. This event is impossible. Why?.

$X = \varnothing$, $n(E) = 0$

$\therefore P(E) = $ zéro

F Let X be the event of getting a positive integer less than 25. This event is certain. Why?

$X = \{1, 2, 3, ..., 24\}$

$\therefore n(X) = 24 = n(E)$

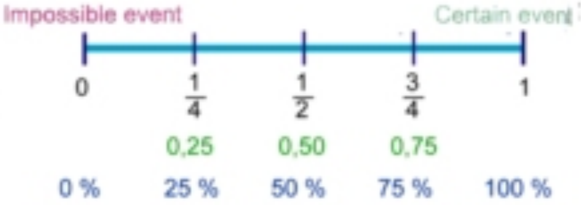
$$P(X) = \frac{n(X)}{n(E)} = \frac{n(E)}{n(E)} = 1$$

Refer to the given example:

1. Impossible event (∅): an event can not be occured.
   The probabity of an impossible event = zero
2. Certain event (S): an event whose outcomes are all possible outcomes
   The probability of a certain event = 1

As illustrated in the opposite figure, where P(A) $\in [0, 1]$

It is possible to write the probability as a fraction, decimal or percentage.



Practice

1 Selecting randomly a card out of 40 similar cards in a box numbered from1 to 40. **Find the probability** of getting a card carries:

- **A** an even number.
- **B** a number is divisible by 3.
- **C** a number is not divisible by 10.
- **D** an even number is divisible by 3.
- **E** a prime number is less than 20.

2 Drawing randomly a colored marble out of a box containing 12 red marbles, 18 white marbles and 20 blue marbles.

Find the probability of selecting:

- **A** a white marble.
- **B** a red marble.
- **C** a yellow marble.
- **D** a non - red marble.
- **E** a red or blue marble.

Exemple (2)

In a survey of favorite weight of a package of wash powder. The manufacturin company asked a group of 300 Ladies using this product. The following table lists th results:

Weight (in gm)	125	250	375	500	Sum
Number of ladies	120	45	96	39	300

I: Selecting randomly a lady, what is the probability to choose:

- **A** 125 g
- **B** 250 g
- **C** 375 g
- **D** 500 g

II: What is your advice to the manager of this company according to the resul of this survey?

Solution

First:

A. The probability of choosing 125 gm $= \frac{120}{300} = \frac{40}{100} = \frac{2}{5} = 0.4 = 4\ \%$

B. The probability of choosing 250 gm $= \frac{45}{300} = \frac{15}{100} = \frac{3}{20} = 0.15 = 15\ \%$

C. The probability of choosing 375 gm $= \frac{96}{300} = \frac{32}{100} = \frac{8}{25} = 0.32 = 32\ \%$

D. The probability of choosing 500 gm $= \frac{39}{300} = \frac{13}{100} = 0.13 = 13\ \%$

Note that:

① It is possible to write the probability in the form of a fraction, a decimal or a percent. For instance, if the probability $= \frac{3}{20}$ then the probability $= \frac{3}{20} \times (100)\ \% = 15\%$

Second: Write down your advices to the manager of the company, discuss it with your classmates and keep the report in your portfolio file.

Practice

The following table shows the results of a survey of favorite transportation means to go to school.

Transportation means	Bus	Private car	Bicycle	on foot
Number of students	3	12	24	66

Selecting randomly a student. Find the probability in percent of choosing:

A. a bus user

B. a bicycle user.

C. a private car user.

D. on foot walker.

Exemple (3)

A life insurance company has found in a sample of 10000 men, between 40 and 50 years old, 67 are dead in one year.

A. What is the probability of a man to die between 40 and 50 years old in one year?

B. Why are these results important for life insurance companies?

C If the company signed life-insurance contract with 50000 men between 40 and 50 years old, then how many death-benefits should be paid in one year.

Solution

A Death probability = $\frac{67}{10000}$ = 0.0067

B Life-insurance companies are interested in experimental probability to find the insurance- rate (instalment).

C The estimated number of death-cases in one year = death probability × number of insured persons = 50000 × 0.0067 = 335

Practice

In producing 300 electric lamps, 18 units found defective.

A What is the probability of a unit to be a defective unit?

B What is the probability of a functional unit?

C Is it possible for a unit to be a functional unit and out of order unit in the same time?

D Find the sum of the probability of a defective unit and the probability of a functional unit. What do you observe?

E If a daily production of this factory was 1600 electric lamps. Find the number of the functional units in that day.

General Exercices

1 During the training of a soccer-team for the final-match of world-cup, a player scored 12 goals out of 15 kicks and another player scored 9 goals out of 12 kicks. whom will the coach choose to kick the penalties during the match? Why?

2 A calculator manufacturing company examined randomly electronic-circuits in a sample of 200 units. The defective production was 6%.

How many units are out of order in this sample?

A If the total production in one month was 1500 units. How many units are
B functional units for marketing?

3 A garment factory produces two types of shirts. The factory made a survey to adjust the production quantity according to market requirments. Samples of 100 shirts are chosen from 5 shopping centres of the factory. The following table lists the results.

Number of shopping center	(1)	(2)	(3)	(4)	(5)
Sold amount of first type	39	82	34	22	53
Sold amount of second type	61	18	66	78	47

A Which type is more demanded? what is the advice you give to the company?

B If the total production of this factory was 4000 shirts. What is your estimated number of shirts of the first type?

4 The following table shows the evaluation of 50 students in one month:

A student is randomly selected. What is the probability of getting a score of:

Estimate	number
Excellent	6
Very good	9
Good	11
pass	16
Fail	8

A Excellent

B Good

C Fail

D less than good

5 A team plays 30 matches in a national league. Its draw probability is 0.3, and its win probability is 0.6.

Find:

A Estimated number of draw matches.

B Estimated number of lose matches.

6 A garment factory in the Tenth of Ramadan city produces 6000 unit daily. As a sample of 1000 units examined, 20 defective units were found. Calculate the number of defective units.

Activites

A survey has been conducted on 100 students about their favourite games which they practice. The result was as follows:

1. Find the probability if a student prefers

   A. Practicing football.
   B. Practicing handball.
   C. Practicing athletics.
   D. Practicing tennis.
   E. Practicing hockey.

favourite game	Number of students
Football	44
Handball	27
athletics	12
Tennis	4
Hockey	13

2. If the number of students is 600. How many students are predicted to practice Hockey?

Unit Test

1. In a fruit packing plant, 30% of fruits is not suitable for exporting because the size is too small. How many tons can be exported in 10 days, if 20 tons of fruits are delivered back daily to the factory?

2. Drawing randomly a colored marble out of a bag containing 32 similar marbles colored red, white, green and yellow. The probability of getting a red marble is $\frac{3}{8}$. Estimate how many red marbles are in the bag?

UNIT FOUR

4

Areas

Unit FOUR

Lesson One

Equality of Areas of Two Parallelograms

Think and Discuss

You will learn

- Relation between areas of two parallelograms.
- Relation between area of a parallelogram and area of a rectangle.
- To calculate area of a parallelogram.
- Relations between a parallelogram and a triangle with common base and drawn between two parallel lines
- To calculate the area of a triangle.

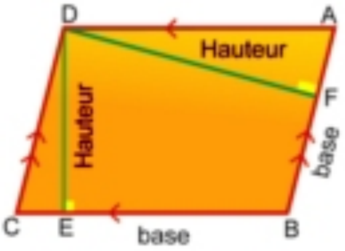
Key-Terms

- Area .
- Parallelogram.
- Rectangle .
- Triangle .
- Base.
- Altitude .
- Two Parallel lines.

Use what you have learned before to find answers of the following questions:

- What is the definition of a parallelogram?
- What are the properties of a parallelogram?
- Is the distance between two parallel lines constant? Explain and give examples of real-life situations.
- Are rectangles, rhombuses and squares special cases of parallelograms? Why?

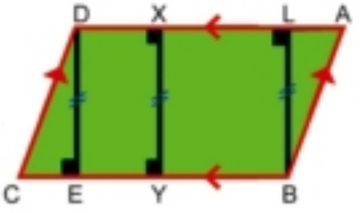
The altitude of a parallelogram:



In the opposite figure: A B C D is a parallelogram. If we consider $\overline{BC}$ as a base and if $\overline{DE} \perp \overline{BC}$, then the length of $\overline{DE}$ **is the corresponding altitude of the base** $\overline{BC}$ **.**

If we consider $\overline{AB}$ as a base of the parallelogram and if $\overline{DF} \perp \overline{AB}$

then the length of $\overline{DF}$ **is the corresponding altitude of the base** $\overline{AB}$.

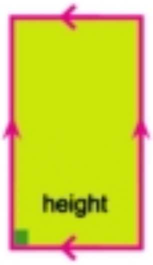


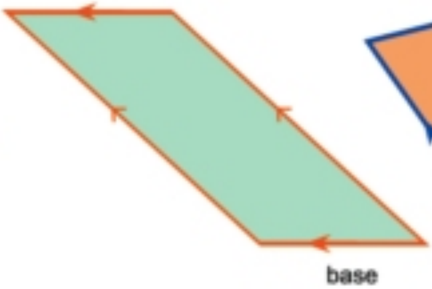
Note that: The altitude of the parallelogram corresponding to the base $\overline{BC}$ is congruent to $\overline{DE}$ where:

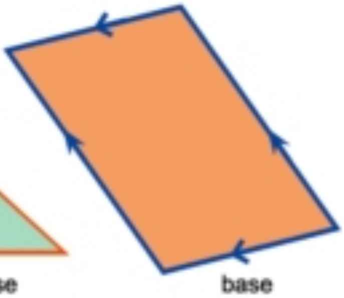
$DE = xy = BL$ why?

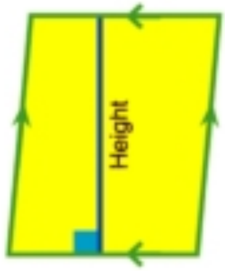
Practice

Determine a base and the corresponding altitude in each case of the following parallelograms:



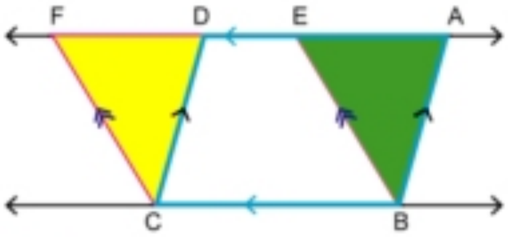


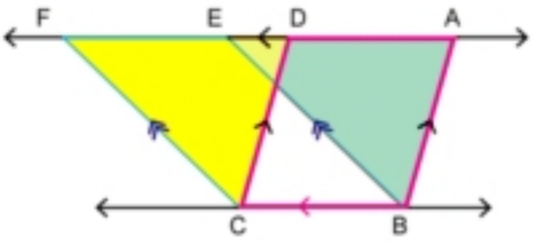




Theorem 1

Surfaces of two parallelograms with common base and between two parallel straight lines, one carrying the base, are equal in area.





Given: A B C D and E B C F are two parallelograms with a common base $\overline{BC}$ and $\overleftrightarrow{BC} \parallel \overleftrightarrow{AF}$

R.T.P.: **area** ▱ A B C D = area ▱ E B C F

Proof: ∵ △ D C F is the image of △ A B E Translation of magnitude BC in the direction of $\overrightarrow{BC}$

∴ △ D C F ≡ A B E Translation is isometry

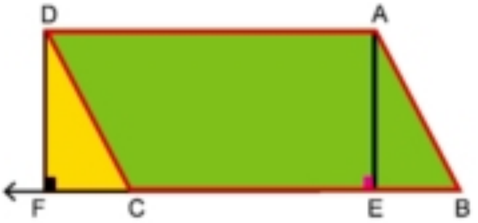
∴ area of figure A B C F - area of △ D C F = area of figure A B C F - area of △ A B E

∴ area of ▱ A B C D = area of ▱ E B C F

(Q.E.D.)

Let's think

In the opposite figure:



▱ABCD, $\overline{AE} \perp \overline{BC}$

If $\overline{DF} \perp \overrightarrow{BC}$, then △ DCF is the image of △ ABE by translation with magnitude .... in the direction of ....

What is the relation between area ▱ ABCD **and area of rectangle** AEFD?

Corollaries

Corollary 1

Parallelogram and rectangle with common base and between two parallel straight lines are equal in area.

Note that:

area of rectangle = length × Width

area of rectangle AEFD = EF × AE = BC × AE why?

Thus, area of ▱ ABCD = BC × AE

Corollary 2

Area of the Parallelogram = length of the base × Corres ponding height

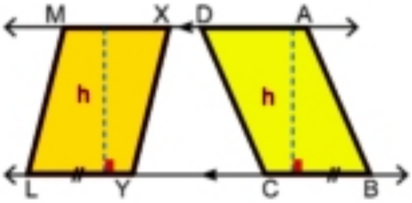
Note that:

The distance between two parallel lines is always constant.

If BC = YL, then

area ▱ ABCD = BC × ......

area ▱ XYLM = YL × ....



What can you conclude?

Corollary 3

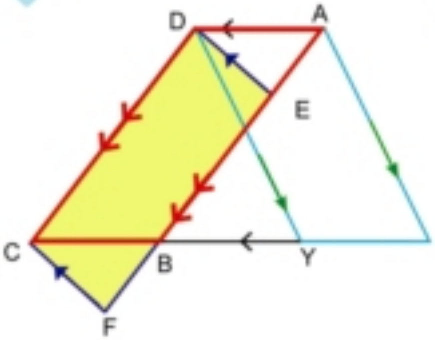
Parallelograms with bases equal in length and lying on a straight line, while the opposite sides to these bases are on another straight line are equal in area.

Practice

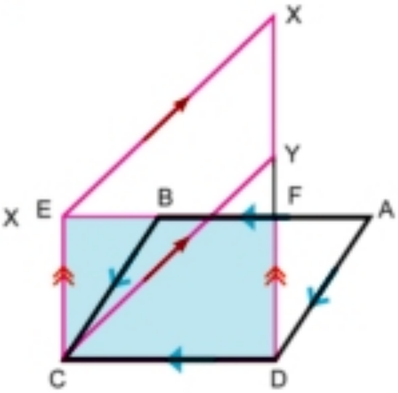
1 In the following figures:

Show that all the three parallelograms have equal areas.

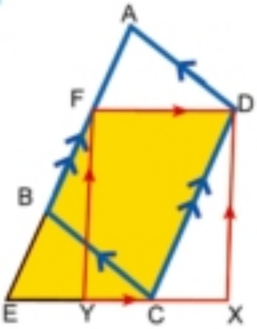
A



B

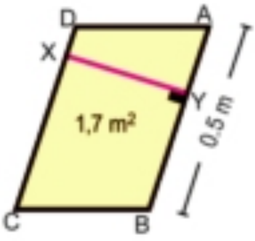


C

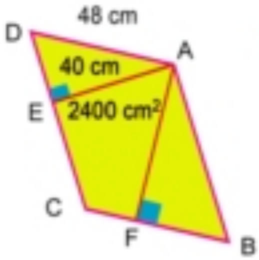


2 **Complete**

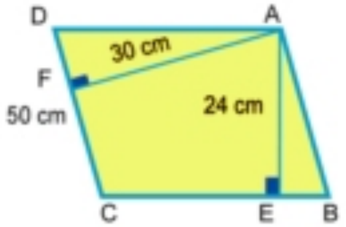
A



B



C



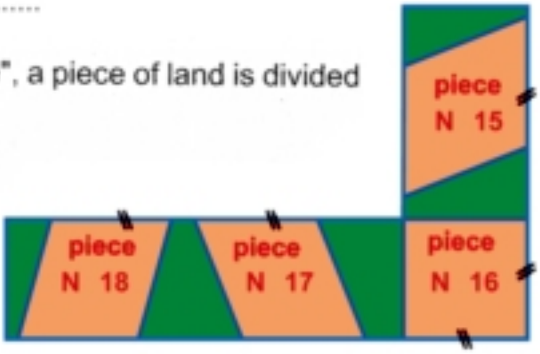
XY = ...

D C = ...

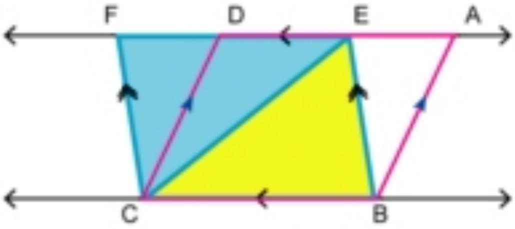
A F = ...

B C = ...

3 In the national project "Build up your home", a piece of land is divided as illustrated in the opposite figure:
Is the area of piece number 15 = the area of the pieces number 16?
State the number of piece of equal areas. Explain your answer.



Let's think



In the opposite figure: $\overleftrightarrow{BC} \parallel \overleftrightarrow{AF}$,

ABCD, and EBCF are two Parallelograms

$\overline{EC}$ is a diagonal in Parallelogram EBCF

∴ **area** △ EBC = ... area of Parallelogram EBCF

∴ **area** Parallelogram EBCF = area ....

∴ **area** △ EBC = ... area of Parallelogram ABCD

Corollary 4

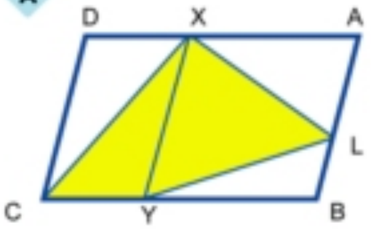
Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them

Practice

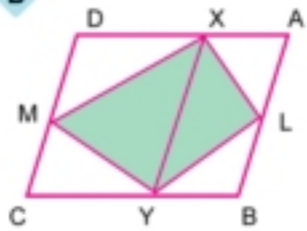
In the following figures $\overline{XY} \parallel \overleftrightarrow{AB}$ **:**

Show that the shaded area is equal to the half of the area of Parallelogram ABCD

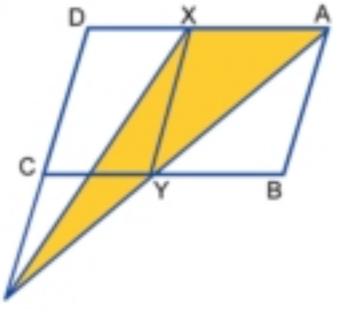
A



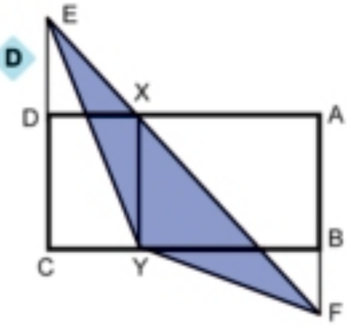
B



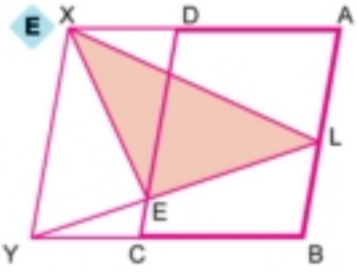
C



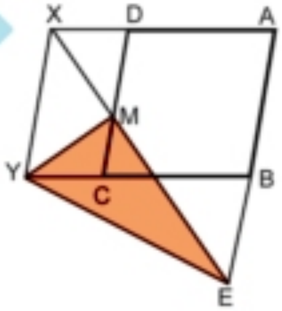
D



E

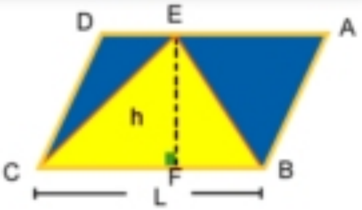


F



Unit 4 : Lesson 1

Let's think



In the opposite figure:
ABCD is a Parallelogram
area of traingle E B C = area of Parallelogram A B C D
= ....×..............×..............

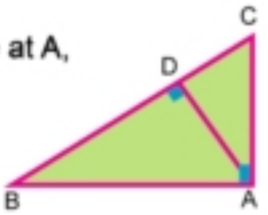
Corollary 3

Area of the Triangle = $\frac{1}{2}$ of the length of the base × its Height

Note that:

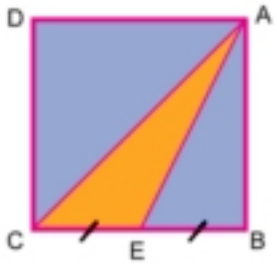
1. The height of a triangle is the length of the perpendicular line segment drawn from a vertex to the opposite side.
2. All perpendiculur line segments of a triangle intersect in one point.

Practice



1. **In the opposite figure:** △ A B C is a right angled triangle at A, $\overline{AD} \perp \overline{BC}$

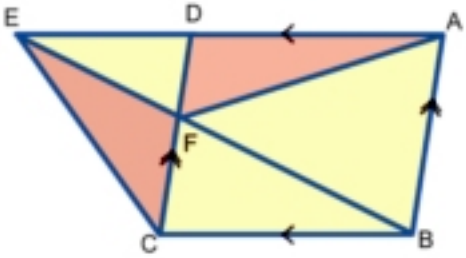
Complete:
area of the triangle A B C = $\frac{1}{2}$ A B ×
area of the triangle A B C = $\frac{1}{2}$ B C ×
∴ A B × = B C ×
Let A B = 4cm and A C = 3cm. What is the length of $\overline{AD}$?



2. **In the opposite figure:** A B C D is a square with perimeter = 24cm,

E is the midpoint of $\overline{BC}$
Complete:
A B = cm, C E = cm
area of the triangle A E C = cm^2

Example



In the opposite figure:

$ABCD$ is a parallelogram , $E \in \overrightarrow{AD}$,

$\overline{BE} \cap \overline{CD} = \{F\}$

Prove that: area of the triangle AFD = area of the triangle EFC

Solution

Given: ▱ $ABCD$, $\overline{BE} \cap \overline{CD} = \{F\}$

R.T.P : area of $\triangle AFD$ = area of $\triangle EFC$

Proof : $\because$ area of $\triangle AFB = \frac{1}{2}$ area of ▱ $ABCD$ (corollary)

$\therefore$ area of $\triangle AFD$ + area of $\triangle BFC = \frac{1}{2}$ area of ▱ $ABCD$ **(1)**

$\because$ area of $\triangle EBC = \frac{1}{2}$ area of ▱ $ABCD$ (corollary)

$\therefore$ area of $\triangle EFC$ + area of $\triangle BFC = \frac{1}{2}$ area of ▱ $ABCD$ **(2)**

from (1) and (2), we have:

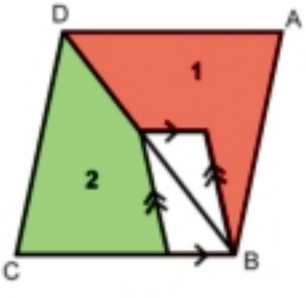
area of $\triangle AFD$ = area of $\triangle EFC$ (Q.E.D.)

Let's think

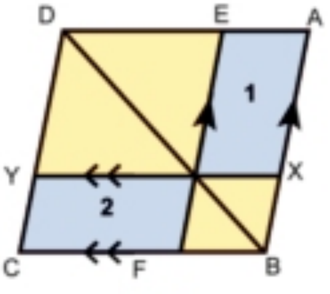
In both figures A and B :

$ABCD$ is a parallelogram.

Why is area of Fig.(1) = area of Fig. (2) ?



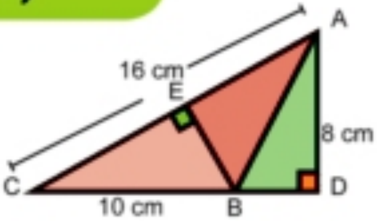
(A)



(B)

Exercices (4 - 1)

1 **In the opposite figure:**



$\overline{AD} \perp \overrightarrow{CB}$, $\overline{BE} \perp \overline{AC}$, $AC = 16$cm.

$BC = 10$cm, $AD = 8$cm.

Find: (**a**) area of $\triangle ABC$ (**b**) **the length of** $\overline{BE}$

2 **In the opposite figure:**

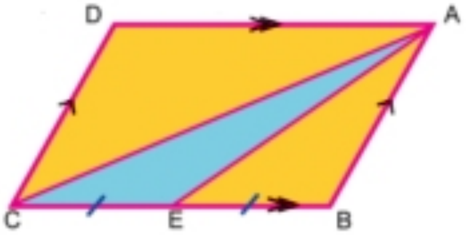
$ABCD$ is a parallelogram with perimeter = 48cm, $BC = 2AB$,

area of $\triangle ABC = 56\text{cm}^2$,

E is the midpoint of $\overline{BC}$. **Find:**

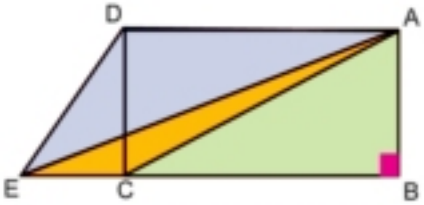
a) The two heights of ▱ ABCD

b) area of $\triangle AEC$



3 **In the opposite figure:** ABCD is a rectangl

Prove that area of $\triangle DAE$ = area of $\triangle ABC$



4 **In the opposite figure:**

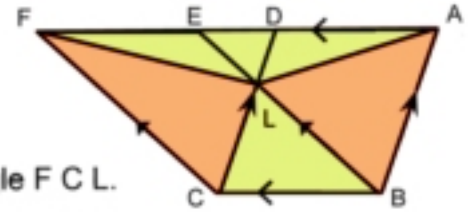
ABCD, EBCF are two parallelograms

$\overline{BE} \cap \overline{CD} = \{L\}$, $D \in \overline{AF}$, $E \in \overline{AF}$

Prove that:

a) area of the triangle ABL = area of the triangle FCL.

b) area of the figure ABCL = area of the figure FCBL

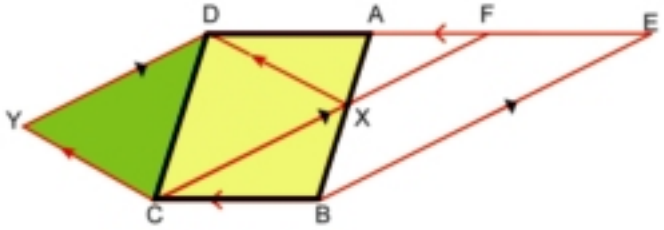


5 **In the opposite figure:**

$\overleftrightarrow{ED} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{XD} \parallel \overleftrightarrow{CY}$,

$\overleftrightarrow{EB} \parallel \overleftrightarrow{FC} \parallel \overleftrightarrow{DY}$, $X \in \overline{FC}$,

$F \in \overline{ED}$ and $A \in \overline{E[}$



Prove that: area of ▱ EBCF = area of ▱ ABCD = area of ▱ DXCY.

Lesson TWO

Equality of the Areas of Two Triangles

Think and Discuss

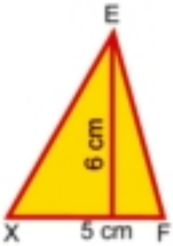
You will learn

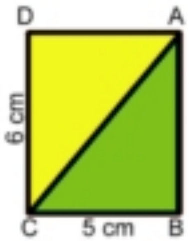
- Relation between the areas of two triangles.

Key-Terms

- Area of a Triangle

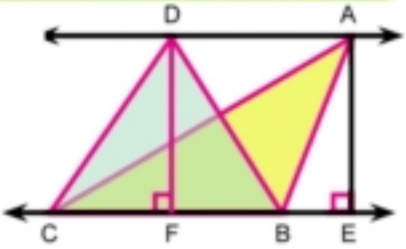
When two triangles are congruent, can you say that they have equal areas? When two triangles have equal areas, can you say that they are congruent?





Theorem 2

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.



Given: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\triangle ABC$ and $\triangle DBC$ have the common base $\overline{BC}$.

R.T.P : area of $\triangle ABC$ = area of $\triangle DBC$

Construction: Draw $\overline{AE} \perp \overleftrightarrow{BC}$ and $\overline{DF} \perp \overleftrightarrow{BC}$

Proof: $\because \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AE} \perp \overline{BC}$ and $\overline{DF} \perp \overleftrightarrow{BC}$

$\therefore$ A E F D is a rectangle, $AE = DF$

$\because$ area of $\triangle ABC = \frac{1}{2} BC \times AE$ **(1)**,

area of $\triangle DBC = \frac{1}{2} BC \times DF = \frac{1}{2} BC \times AE$ **(2)**

From (1) and (2), we have

$\therefore$ area of $\triangle ABC$ = area of $\triangle DBC$

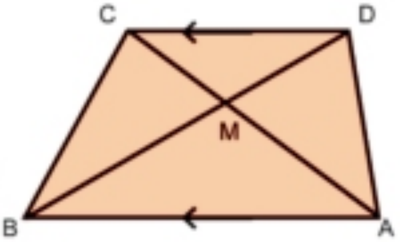
(Q.E.D.)

Practice

1 **In the opposite figure:**

$\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

Complete and justify each step of your answer:



A area of $\triangle ADB$ = area because

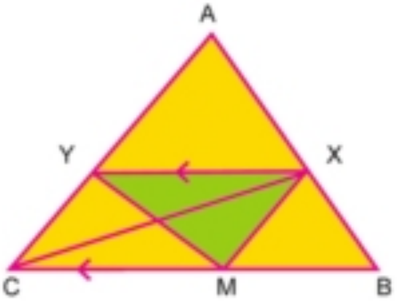
B area of $\triangle DAC$ = area because

C area of $\triangle DAM$ = area because

2 **In the opposite figure:**

$\triangle ABC$, $X \in \overline{AB}$, $Y \in \overline{AC}$,

$\overline{XY} \parallel \overleftrightarrow{BC}$, $M \in \overleftrightarrow{BC}$

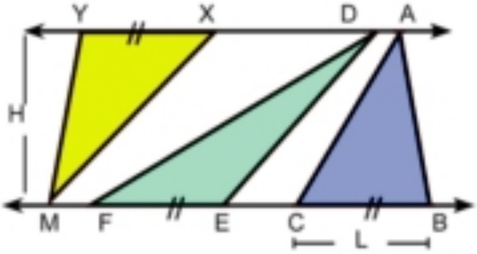


Complete: area of $\triangle XMY$ = area

area of figure AXMY = area Why?

Corollaries:

1 Triangles of bases equal in length and lying between two parallel straight lines are equal in area.



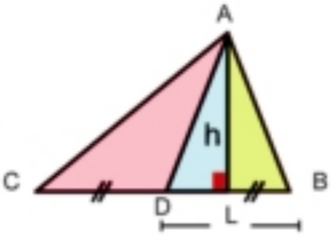
Note that:

$\overleftrightarrow{AY} \parallel \overleftrightarrow{BC}$, $BC = EF = xy$

$\therefore$ area of $\triangle ABC$ = area of $\triangle DEF$ = area of $\triangle XYM = \frac{1}{2}$ L.H

2 The median of a triangle divides its surface into two triangular surfaces equal in area.

Note that:

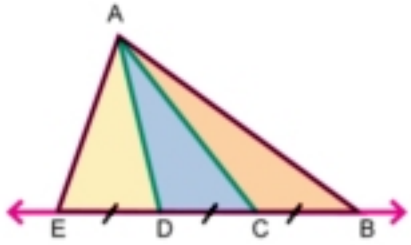


$\overline{AD}$ is a median of $\triangle ABC$ (BD = DC = L)

$\therefore$ area of $\triangle ABD$ = area of $\triangle ADC = \frac{1}{2}$ L × h

③ Triangles with congruent bases on one straight line and have a common vertex are equal in area.

area of △ A B C = area of△ A C D = area of △ A D E



Practice

△ A B C with a median $\overline{AD}$, E ∈ $\overline{AD}$, draw $\overline{BE}$ and $\overline{CE}$

Prove that : area of △ A B E = area of △ A C E

Complete

∵ $\overline{AD}$ is a median in the triangle ....

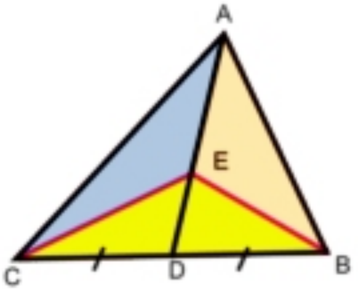
∴ area of △ A B D = area .... (1)

∵ .... is a median in △ E B C

∴ area of △ E B D = area .... (2)

subtracting (2) from (1) , then

area of △ A B E = ...



Exemple :

In the opposite figure:

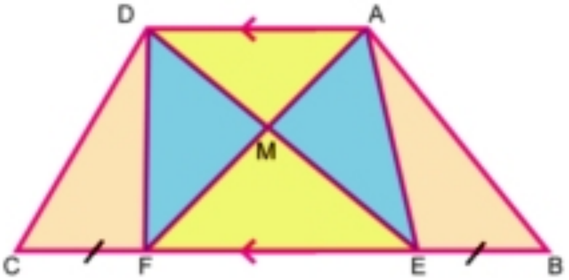
$\overline{AD}$ // $\overline{BC}$, E ∈ $\overline{BC}$, F ∈ $\overline{BC}$ where

B E = C F , $\overline{AF}$ ∩ $\overline{ED}$ = {M}

prove that :

first: area of△ A ME = area of△ D M F

Second: area of the figure A B E M = area of the figure D C F M



Proof:

∵ $\overline{AD}$ // $\overline{EF}$, and △ A E F and △DEF have a common base $\overline{EF}$

∴ area of △ A E F = area of △ D E F

subtracting area of △ ME F from both sides, then

area of △ A E M = area of △ D F M (1) (I.Q.E.D)

$\because BE = CF$, $\overline{AD} \parallel \overleftrightarrow{BC}$

$\therefore$ area of $\triangle ABE$ = area of $\triangle DCF$ (2)

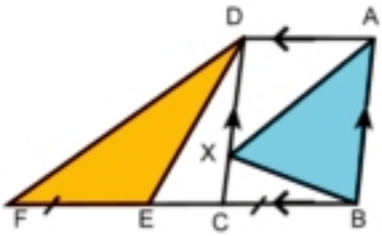
Adding (1) and (2) **we have:**

area of the figure ABEM = area of figure DCFM (Q.E.D)

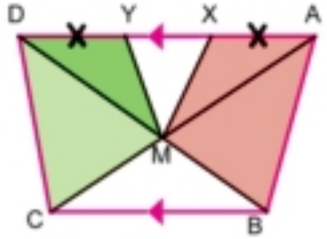
Practice

Show that all the shaded figures have equal areas (Use given information):

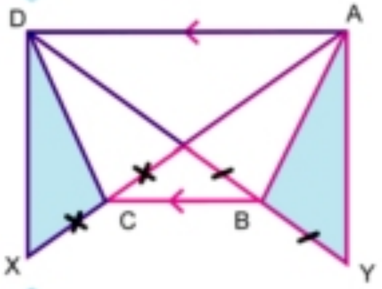
A



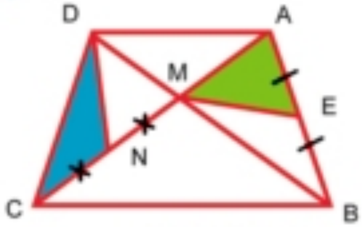
B



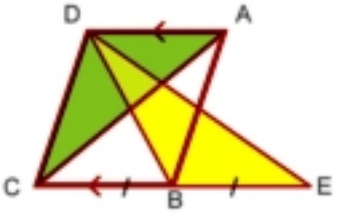
C



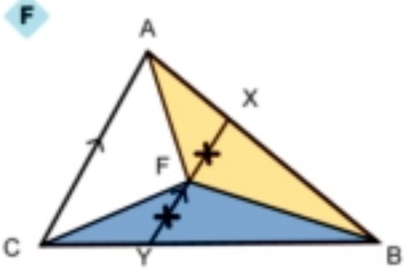
D



E



F



Theorem 3

If two triangles are equal in area and drawn on the same base and in one side of it, then their vertices lie on a straight line parallel to this base.

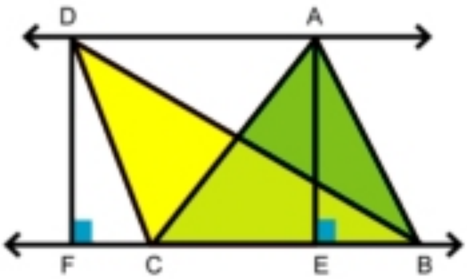
Given: area of $\triangle ABC$ = area of $\triangle DBC$.

$\overline{BC}$ is a common base

R.T.P: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

Construction:

Draw $\overline{AE} \perp \overleftrightarrow{BC}$, $\overline{DF} \perp \overleftrightarrow{BC}$



Proof: ∵ area of $\triangle ABC$ = area of $\triangle DBC$

$\therefore \frac{1}{2} BC \times AE = \frac{1}{2} BC \times DF$

$\therefore AE = DF$

$\because \overline{AE} \perp \overleftrightarrow{BC}$, $\overline{DF} \perp \overleftrightarrow{BC}$

$\therefore \overline{AE} \parallel \overline{DF}$

∴ Figure AEFD is a rectangle Thus : $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

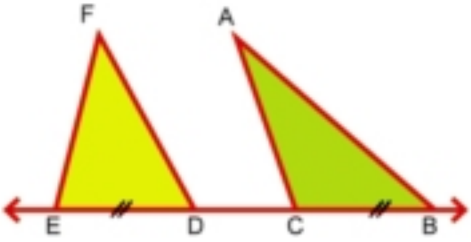
Let's think

1 In the opposite figure:

B, C, D, and E are collinear , where

BC = DE

If area of $\triangle ABC$ = area of $\triangle FDE$. What can you conclude ? Explain your answer.



2 In the opposite figure: $D \in \overleftrightarrow{BC}$, $A \in \overleftrightarrow{FE}$

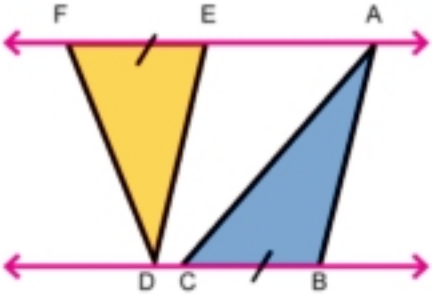
BC = EF

If:

area of $\triangle ABC$ = area of $\triangle DEF$

What can you conclude? Explain your answer.

Note that: $\overleftrightarrow{AF} \parallel \overleftrightarrow{BC}$. Why?

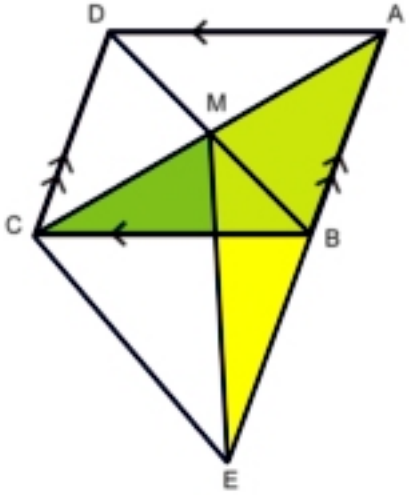


Exemple

ABCD is a parallelogram , $\overline{AC} \cap \overline{BD} = \{M\}$

$E \in \overrightarrow{AB}$ where area of $\triangle AME$ = area of $\triangle ABC$

Prove that : The figure BECD is a parallelogram.



Proof: ∵ area of $\triangle AME$ = area of $\triangle ABC$

Subtracting area of $\triangle ABM$ from both sides

∴ area of $\triangle BME$ = area of $\triangle BMC$

and both triangles have the common base $\overline{BM}$ and in one side of the base $\overline{BM}$.

∴ $\overleftrightarrow{CE} \parallel \overleftrightarrow{BM}$ (1)

∵ The figure A B C D is a parallelogram

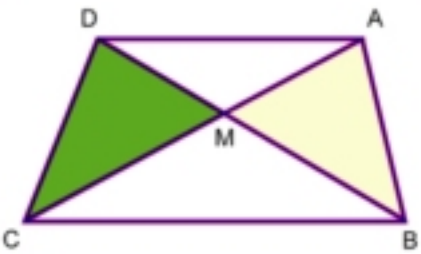
∴ $\overleftrightarrow{BE} \parallel \overleftrightarrow{DC}$ (2)

from (1) and (2) the figure DBEC is parallelograr

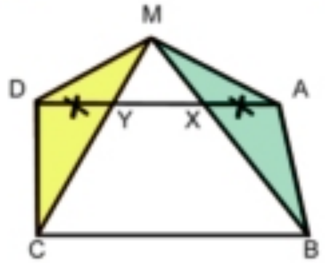
Practice

1 **In the following figures all the colored triangles have the same area . Explain why $\overline{AD} \parallel \overline{BC}$.**

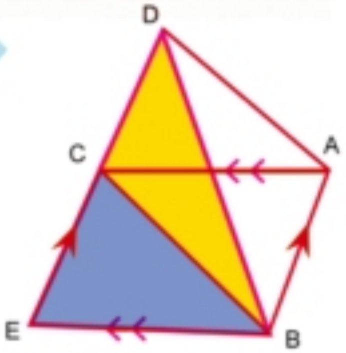
A



B



C



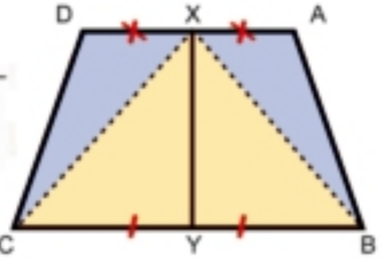
2 **In the opposite figure:**

A B C D is a quadrilateral, where

X is the midpoint of $\overline{AD}$ and Y is the midpoint of $\overline{BC}$

area of the figure ABYX = area of the figure DCYX

Prove that: $\overline{AD} \parallel \overline{BC}$



Problem solving Tip

Draw $\overline{BX}$, $\overline{CX}$

In $\triangle$ X B C, $\overline{XY}$ is a median, what can you conclude?

area of $\triangle$ AXB = area .... why?

$\overline{AD} \parallel \overline{BC}$ Why?

Exercices (4 - 2)

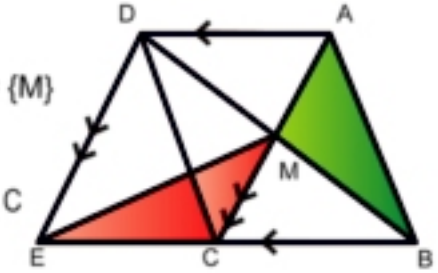
1 **In the opposite figures:**

$\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $E \in \overrightarrow{BC}$, $\overleftrightarrow{AC} \parallel \overrightarrow{DE}$, $\overline{AC} \cap \overline{BD} = \{M\}$

Prove that :

First: area of $\triangle$ A B M = area of $\triangle$ D C M = area of $\triangle$ EM C

second: area of $\triangle$ D B C = area of $\triangle$ E B M

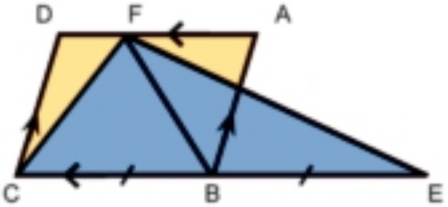


2 **In the opposite figure:**

A B C D is a parallelogram, $E \in \overleftrightarrow{CB}$ where B C = B E

Prove that:

area of $\triangle$ F EC = area of ▱ A B C D



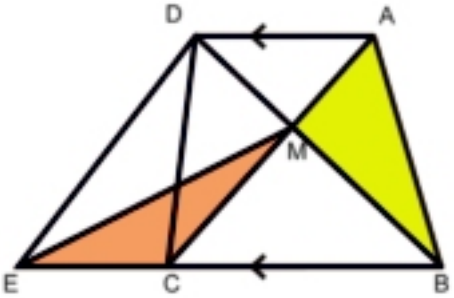
3 **In the opposite figure:**

A B C D is a quadrilateral with $\overline{AD} \parallel \overline{BC}$,

$E \in \overrightarrow{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

area of $\triangle$ A B M = area of $\triangle$ E C M.

prove that: $\overline{DE} \parallel \overleftrightarrow{AC}$



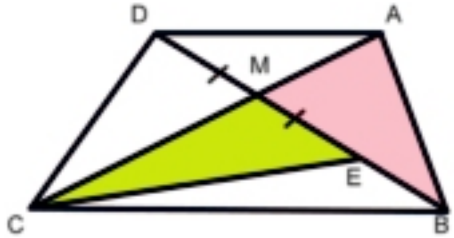
4 **In the opposite figure:**

A B C D is a quadrialteral whose diagonals are intersecting at M,

$E \in \overline{BM}$ where ME = M D,

area of $\triangle$A M B = area of $\triangle$ C ME

prove that: $\overline{AD} \parallel \overline{BC}$



Aeras of Some geometric Figures

Lesson Three

Think and Discuss

You will learn

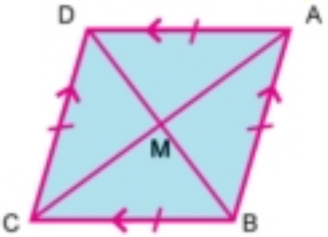
- To find the area of a rhombus.
- To find the area of a square in terms of its diagonal.
- To find the area of a Trapezium.

Key - terms

- Square.
- Rhombus.
- Trapezium
- Area.

You have learned before that the rhombus is a parallelogram whose sides are equal in length.

- What is the Relation between the diagonals of the rhombus?
- How can you calculate the area of the rhombus?

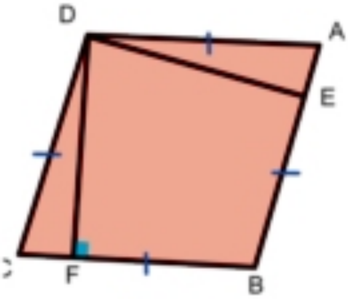


Area of the rhombus:

① If the side length of a rhombus is b and its height is h, then
Area of Rhombus = b × h
i.e:

area of the rhombus

= base length × **height**



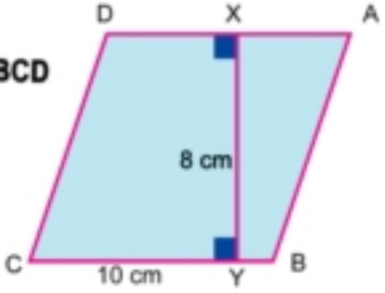
Let's think

is D E = D F? Explain.

Practice

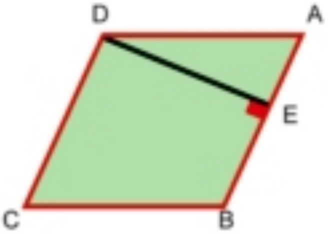
1 find the area of the rhombus ABCD

A area = ...



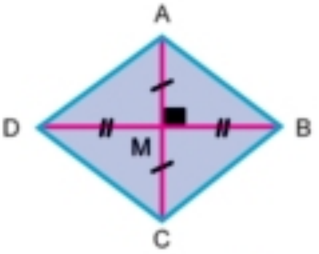
B Perimeter of rhombus A B C D = 24cm, D E = 5cm

area =



2 **You know:**

The diagonals of the rhombus are perpendicular and bisecting each other. Refres to the opposite figure, complete:



area of rhombus A B C D $= 2$ area $\triangle ABD$

$= 2 \quad \times \frac{1}{2} BD \times$

$= \frac{1}{2} \quad \times BD \quad \times 2$

$= \frac{1}{2} BD \times$

Area of the rhombus = half of the product of the lengths of its diagonals.

a **The Square is a rhombus whose diagonals are equal in length.**

Area of the square = $\frac{1}{2}$ of the square of the length of its diagonal

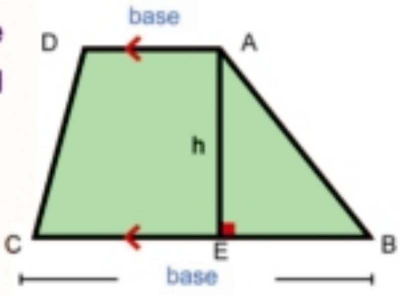
Practice

Find the area of the following figures:

1. A rhombus whose side length is 12cm and whose height is 8cm.
2. A rhombus whose diagonals length are 8cm and 10cm.
3. A square whose diagonal length is 8cm.
4. A rhombus whose perimeter is 52cm and the length of one of its diagonal is 10cm.
5. A rhombus whose perimeter is 60cm and the measure of one of its angles is 60°.

Trapezium

A Trapezium is a quadrilateral whose two opposite sides are parallel. The two opposite sides are called bases and the other two sides are called legs.



In the opposite figure:

$\overline{AD}$, $\overline{BC}$ are bases of the Trapezium A B C D

$\overline{AB}$ and $\overline{DC}$ are legs of the Trapezium A B C D

A Trapezium has only one height which is the perpendicular distance between its bases

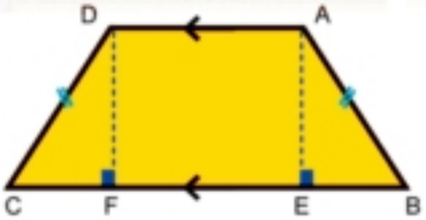
Let's think

Does the diagonal of a trapezium divide it into two triangles with equal areas?

If ABCD is an isosceles Trapezium, in $\overline{AB}$, $\overline{DC}$: is m (∠ B) = m (∠ C)?

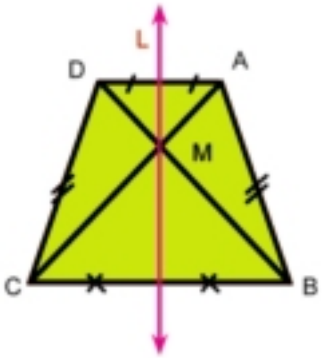
Draw $\overline{AE} \perp \overline{BC}$ and $\overline{DF} \perp \overline{BC}$

Explain your answer.



Isosceles Trapezium:

If: A B C D is a Trapezium with A B = C D, then

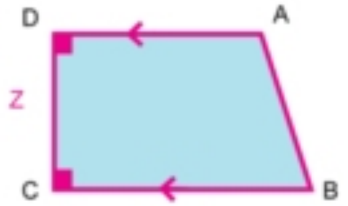


① The base angles are equal in measure.
m (∠ B) = m (∠ C), m (∠ A) = m (∠ D)

② The diagonals are equal in length A C = B D
$\overline{AC} \cap \overline{BD}$ = {M}
∴ A M = D M, B M = C M

③ The isosceles trapezium has only one axis of symmetry (L) which is the perpendicular bisector of its bases.

Right Trapezium



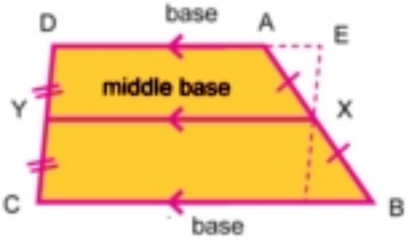
A right Trapezium is a Trapezium whose one of its legs is perpendicular to its two parallel bases

In the opposite figure: $\overline{DC} \perp \overline{BC}$ and $\overline{CD} \perp \overline{AD}$,

$\therefore$ The height of Trapezium = The length of $\overline{CD}$

Middle base of Trapezium

A middle base of a trapezium is a segment $\overline{XY}$ whose endpoints are the midpoints of the non-parallel sides of Trapezium A B C D



Note that:

$\overline{XY} \parallel \overline{BC} \parallel \overline{AD}$

The length of $\overline{XY} = \frac{1}{2}(AD + BC)$

Practice

Find the length of the middle base of a trapezium whose two bases lengths are 7cm and 13cm.

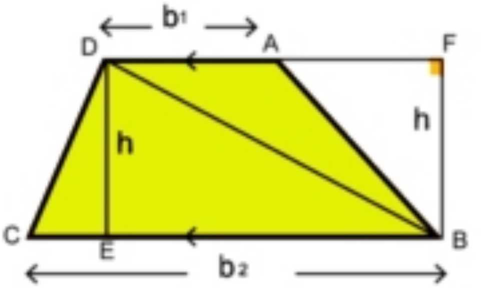
Area of trapezium:

Area of trapezium A B C D = area $\triangle$ A B D + area $\triangle$ D B C

$= \frac{1}{2} AD \times BF \quad + \frac{1}{2} BC \times DE$

$= \frac{1}{2} b_1 h \quad + \frac{1}{2} b_2 \times h$

$= \frac{1}{2}(b_1 + b_2) h$



Area of a Trapezium = half of the sum of lengths of the two parallel bases × height.

Note that: The middle base of the trapezium is parallel to the two bases and its length is equal to half of the sum of their lengths.

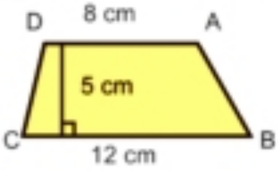
Area of a Trapezium = the length of the middle base × its Height.

Unit 4 : Lesson 3

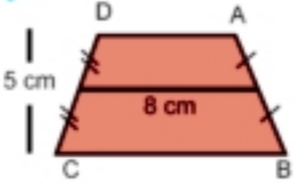
Practice

Find the area of each of the following figurers by using the given data :

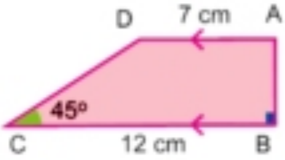
A



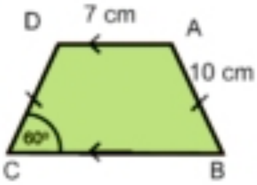
B



C



D



Exercices (4 - 3)

1. Find the height of a trapezium with area of 450cm^2 and the two base lengths are 24cm, and 12cm.

2. The area of a trapezium is 108cm^2 and the length of one of its parallel bases is 15cm. Find the length of the other base, if the height of the trapezium is 8cm.

3. The area of a trapezium is 180cm^2 , its height is12cm. Find the lengths of its parallel bases, if the ratio between their lengths is 3 : 2.

4. Two pieces of land have equal areas, one of them has the shape of a rhombus whose diagonals are 18m and 24m, and the other one has the shape of a trapezium whose height is 12m. Find the length of its middle base.

5. The area of an isosceles Trapezium is 120cm^2 , its perimeter is 60cm and the length of its middle base is 20cm. Find the lengths of its bases.

6. A B C D is a rectangle with AB = 6cm and B C = 8cm. X, Y, L and M are the midpoints of the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$, respectively.
   - **A** Prove that figure X Y L M is a rhombus and find its area.
   - **B** Find the height of the rhombus X Y L M.

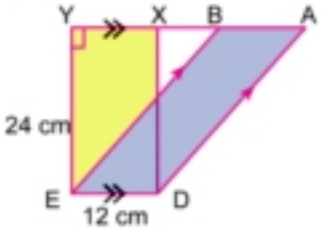
7. A piece of land has the shape of a trapezium whose area is 4000m^2 . The lengths of the two parallel bases and its height are of ratio 3:2:4, respectively. Find the length of its middle base.

General Exercises

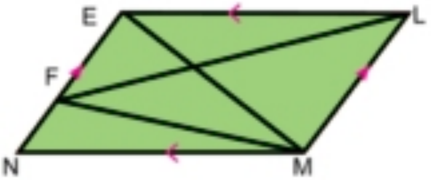
1 **In the opposite figure:** $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$, X and Y $\in \overleftrightarrow{AB}$, XDEY is a rectangle, $\overline{AD} \parallel \overline{BE}$.

a) Find the area of the figure A B E D

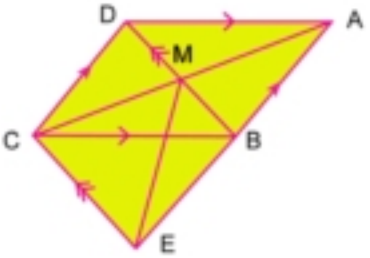
b) If A D = 30cm, then find the perpendicular from B to $\overline{AD}$.



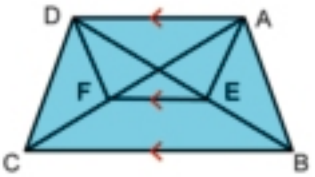
2 In the opposite figure: LMNE is a parallelogram. Prove that the area of $\triangle$ LEF + the area of $\triangle$ MFN = the area of $\triangle$ LEM.



3 A B C D and B E C D are parallelograms where $\overline{AC} \cap \overline{BD} = \{M\}$. Prove that the area of $\triangle$ A B D = the area of $\triangle$ M E C



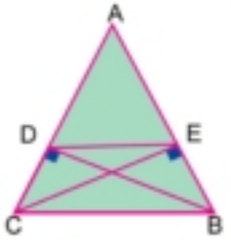
4 **In the opposite figure:** $\overline{AD} \parallel \overline{BC} \parallel \overline{EF}$.
Prove that the area of $\triangle$ A B E = the area of $\triangle$ D C F



5 **In the opposite figure:** A B = A C, $\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

a) Prove that $\overline{ED} \parallel \overline{BC}$

b) The area of $\triangle$ A D B = the area of $\triangle$ A E C



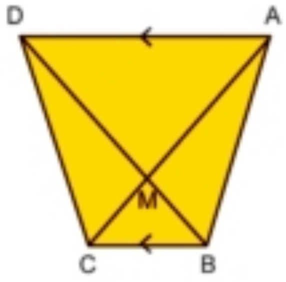
6 **In the opposite figure:** $\overline{AD} \parallel \overline{BC}$

Prove that the area of $\triangle$ A B M = the area of $\triangle$ D M C.

Given the the area of $\triangle$ M B C = 20cm^2,

and the area of $\triangle$ A B M = 3 times the area of $\triangle$ M B C.

Calculate the area of the rectangle drawn on $\overline{BC}$ and its other base is on $\overleftrightarrow{AD}$.

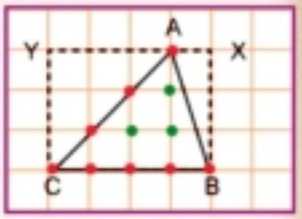


Activites

Use the memory device " **Pick** " for calculating the are

By using a grid. How can you find the area of a polygon

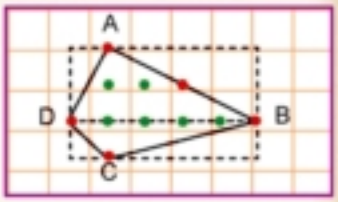
To find an answer, notice the following:



❶ In the opposite figure:

area △ A B C $= \frac{1}{2}$ area of the rectangle X B C Y

$= \frac{1}{2} \times 4 \times 3 = 6$ sq. units

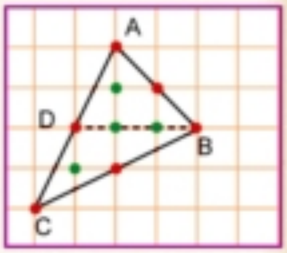


❷ In the opposite figure:

area of figure A B C D = area of △ A B D + area of △ C B D

$= \frac{1}{2} \times 5 \times 2 + \frac{1}{2} \times 5 \times 1$

$= 5 + \frac{5}{2} = 7\frac{1}{2}$ sq. units.



❸ In the opposite figure:

To find the area of △ A B C , we divide the figure into regions whose areas can be calculated.

∴ area of △ A B C = area of △ A B D + area of △ B C D

$= \frac{1}{2} \times 3 \times 2 + \frac{1}{2} \times 3 \times 2$

$= 6$ sq. units.

Refer to the above figures:

Observe the number of red dots that represent the figure on squarers of the grid and they are called boundary points, and they are denoted by P. Also, observe the number of green dots inside the figure and they are called interior dots and are denoted by i. Complete the following table and check your answers.

	i	P	$M = i + \frac{P}{2} - 1$	Area
First Figure	3	8	$3 + \frac{8}{2} - 1$	6
Second Figure	6	5	$6 + \frac{5}{2} - 1$	$7\frac{1}{2}$
Third Figure	....	....	......	.....

Draw other figures on the grid and calculate the area in each case geometrically. Test and check by using the memory device pick.

pick rule Area = **Number of interior points** + $\frac{\text{Number of boundary points of the polygon}}{2} - 1$

Explain how can you use pike in a real-world applications.

Unit Test

1 Complete:

A The area of the rhombus whose diagonals 6cm and 8 cm = ...

B The diagonals of an isosceles trapezium ....

C The area of the trapezium whose middle base 7cm, and height 6cm =.........

D Triangles with congruent bases and drawn between two parallel lines are ....

E The median of a triangle divides its area into ....

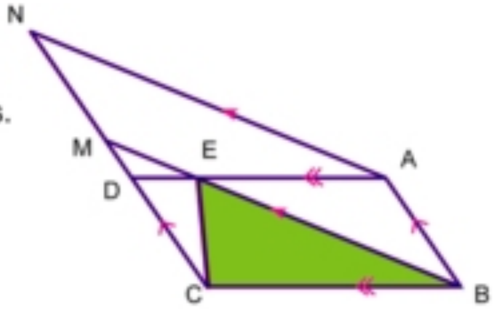
F The area of a square is 50cm^2. The length of its diagonal = ...cm

2 In the opposite figure:

A B C D and A B M N are two parallelograms.

Prove that: the area of

$\triangle$ E B C = $\frac{1}{2}$ the area of ▱ A B M N



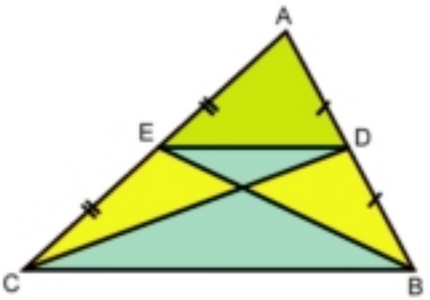
3 In the opposite figure:

$\triangle$ A B C , D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

Prove that:

a) area of $\triangle$ **D** B C = area of $\triangle$ E B C

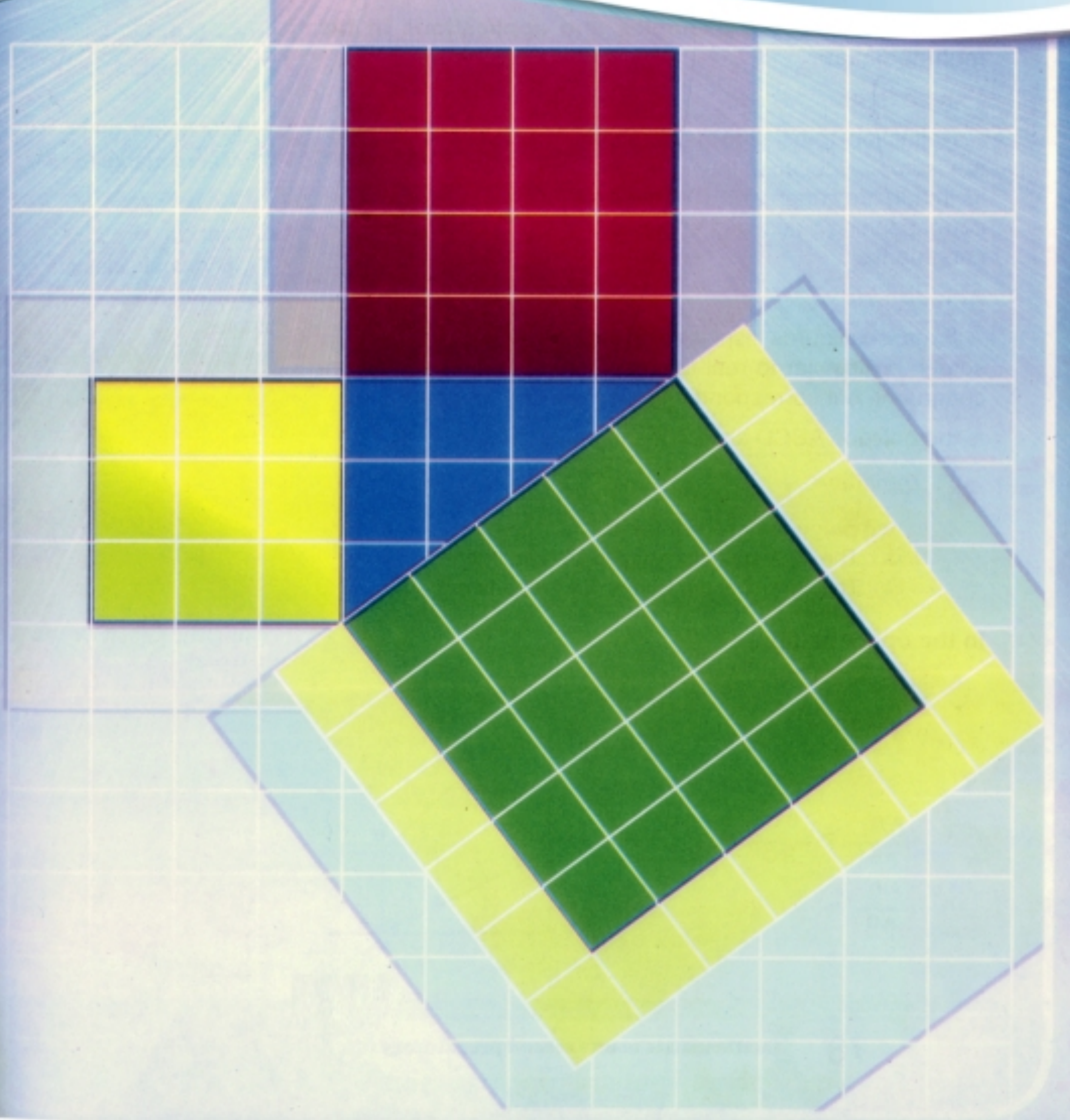
b) $\overline{DE} \parallel \overline{BC}$



UNIT FIVE

5

Similarity and Projections

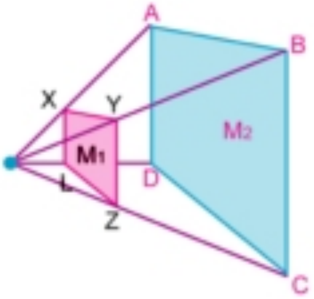


Similarity

Lesson One

Think and Discuss

During displaying examples and applications in the multmedia lab.



Usama Said:

Reflection, translation and rotation are isometry, because the figure and its image are congruent. This means corresponding sides and angles are congruent .

Ahmed Said:

Exercises figures displayed on the screen are similar to real figures. Corresponding angles are congruent, but corresponding sides are proportional.

Is the Polygon ABCD similar to the polygon XYZL? Why?

You will learn

- The concept of similarity.
- Similar of Polygons.
- Similar of triangles.

Key-Terms

- Similar.
- Proportional sides.
- corresponding angles.

Definition

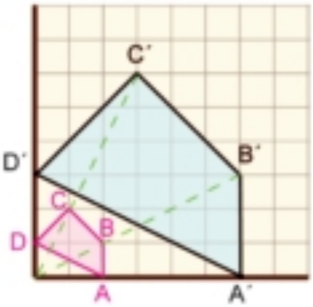
Two polygons are similar if:

- The corresponding angles are congruent
- The corresponding sides are proportional.

In the opposite figure

Exemple :

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{A'D'}{AD} = \frac{3}{1}$$



$m(\angle A') = m(\angle A)$, $m(\angle B') = m(\angle B)$,
$m(\angle C') = m(\angle C)$, $m(\angle D') = m(\angle D)$

The polygon ABCD is similar to the polygon $A'B'C'D'$.

Note that:

① The order of corresponding vertices should be kept in giving names of similar polygons, Similarity is denoted by the sign (~). Fig. $A'B'C'D'$ (~) Fig. ABCD means two similar figures.

② The proportional ratio between corresponding sides is called the ratio of inlargement or drawing scale.

Notice: If the proportional ratio = 1, then the two polygons are congruent.

③ All the regular polygon that have the same number of sides are similar. why?

④ If two polygons are similar, then the corresponding angles are congruent and the corresponding sides are proportional as well.

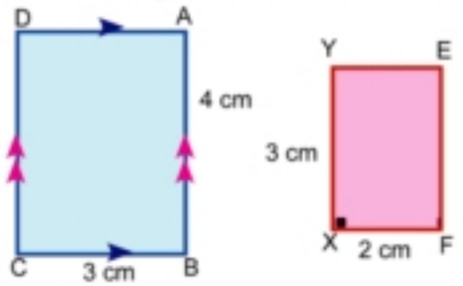
Think: **The square and the rectangle are not similar although the corresponding angles are congruent . Why?**

The corresponding sides of a square and a rhombus are proportional but they are not similar.

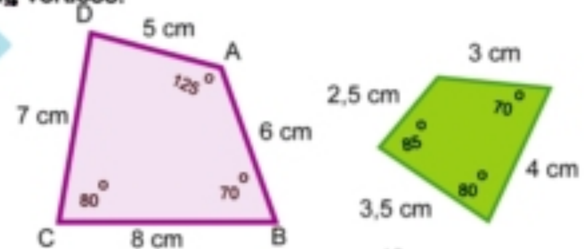
Practice

1 Which of the following pairs of polygons are similar and why? write the similar polygons following the same orders of corresponding vertices.

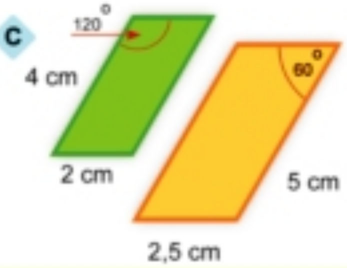
A



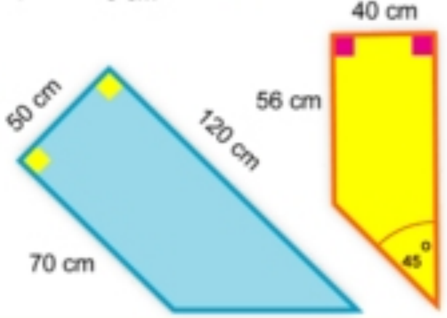
B



C



D



Similarity of two triangles

definition

Two triangles are similar if there exists one of the following conditions :

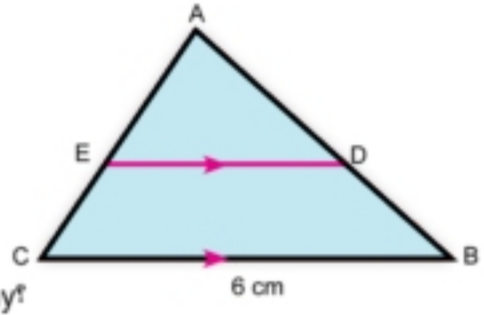
- **The corresponding angles are congruent.**
- **the corresponding sides are proportional.**

Exemple :

In the opposite figure: ABC is a triangle in which AB = 5cm, BC = 6cm, AC = 4cm, and D ∈ $\overline{AB}$ where AD = 3cm, $\overline{DE} \parallel \overline{BC}$, $\overline{DE} \cap \overline{AC} = \{E\}$.

A **Prove that** $\triangle ADE \sim \triangle ABC$.

B **Find the length of** $\overline{DE}$ and $\overline{AE}$



Solution

$\because \overline{DE} \parallel \overline{BC}$

$\therefore m(\angle ADE) = m(\angle B)$, $m(\angle AED) = m(\angle C)$ Why?

$\because \angle A$ is common in $\triangle ADE$ and $\triangle ABC$.

$\therefore \triangle ADE \sim \triangle ABC$ corresponding angles are congruent, So:

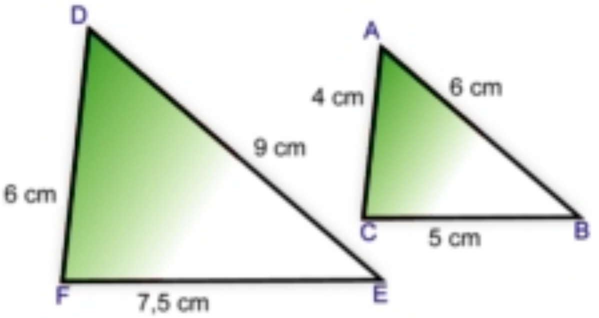
$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \quad \therefore \frac{3}{5} = \frac{DE}{6} = \frac{AE}{4}$$

$$\therefore DE = \frac{3 \times 6}{5} = 3.6\text{cm and } AE = \frac{3 \times 4}{5} = 2.4\text{cm}$$

Practice

Using the given in the opposite figures.

Prove that



A $\triangle DEF \sim \triangle ABC$

B $\frac{\text{Perimeter of } \triangle DEF}{\text{Perimeter of } \triangle ABC}$ = the ratio of the similarity

Note that :

the ratio between the perimeters of two similar triangles

= the ratio between any two of corresponding sides

Exercise (5 – 1)

1. In each of the following figures :

Find the value of X , given that each pair of triangles are similar

(a)

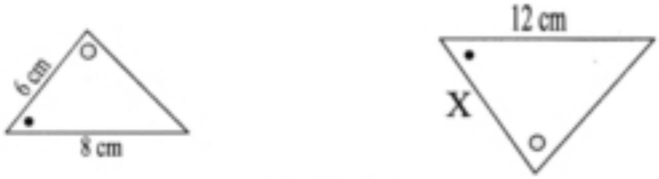


Figure (1)

(b)

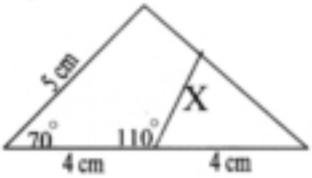


Figure (2)

(c)

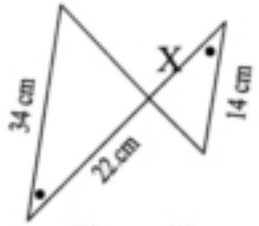


Figure (3)

(d)

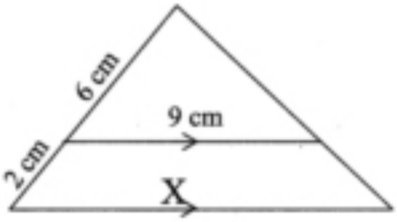


Figure (4)

(e)

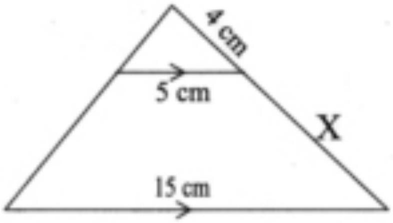


Figure (5)

(f)

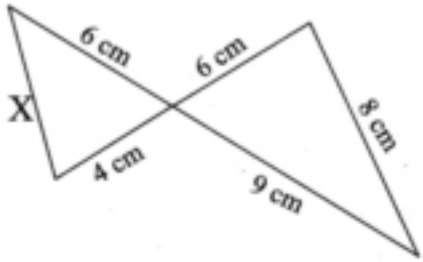


Figure (6)

(g)

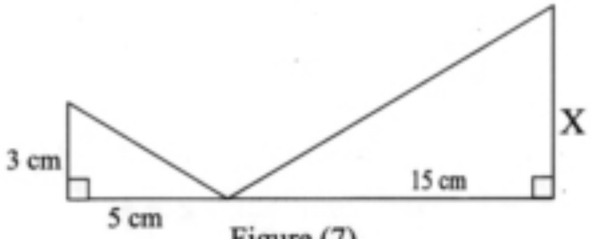
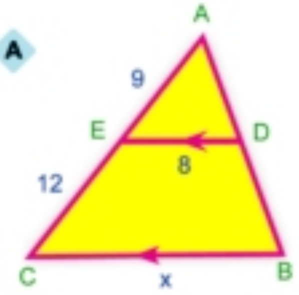


Figure (7)

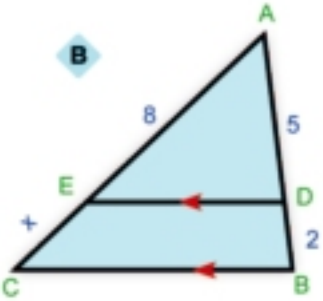
2 **In the following figures: Find the value of X and y (Lengths are estimated in cm)**

:

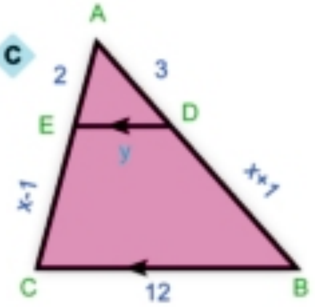
A



B

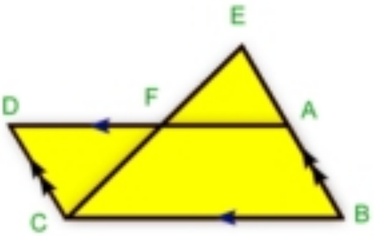


C



3 **In the opposite figure:** ABCD is a parallelogram, where $E \in \overrightarrow{BA}$, $\overrightarrow{CE} \cap \overline{AD} = \{F\}$, BC = EC = 10cm, AB = 4cm, and FD = 6cm.

Find the length of: $\overline{EB}$, $\overline{EA}$ and $\overline{FC}$

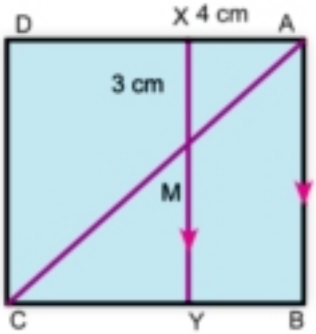


4 **In the opposite figure:** ABCD is a rectangle, in which AD = 12cm,

$X \in \overline{AD}$ where A X = 4cm, $\overline{XY} \backslash\backslash \overline{AB}$ and intersects $\overline{AC}$ at M and $\overline{BC}$ at y,

where M X = 3cm.

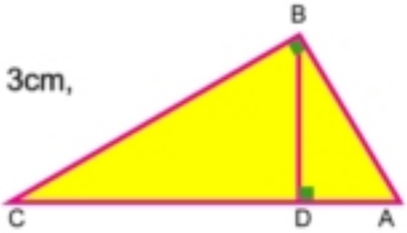
- **A** Prove that $\triangle AMX \sim \triangle CMY$.
- **B** Find the perimeter of $\triangle YMC$.
- **C** Is the figure A B Y M ~ the figure C D X M? Why?



5 A B C is a right angled triangle at B , in which AB = 3cm, B C = 4cm, and $\overline{BD} \perp \overline{AC}$.

Prove that : $\triangle BAC \sim \triangle DAB$,

then find the length of $\overline{AD}$, $\overline{DC}$.



Converse of Pythagoras Theorem

Lesson Two

Think and Discuss

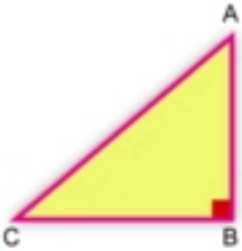
You will learn

- Converse of pythagoras theorem.
- Using pythagoras theorem on solving problems.

You know from Pythagoras Theorem that if △ ABC is a right angled triangle at B, then

$$(AC)^2 = (AB)^2 + (BC)^2$$

Now, we will learn the converse of the pythagorean Theorem.

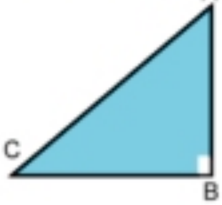


Converse of Pythagoras Theorem:

In a triangle if the sum of the areas of two squares on two sides is equal to the area of the square on the third side, then the angle opposite to this side is a right angle.

i.e. in △ ABC, if : $(AB)^2 + (BC)^2 = (AC)^2$

then : m (∠B) = 90° and △ ABC is a right angled triangle at B.

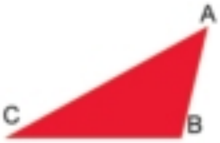


The converse of Pythagoras Theorem can be rewritten as:

In a triangle if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

Corollary:

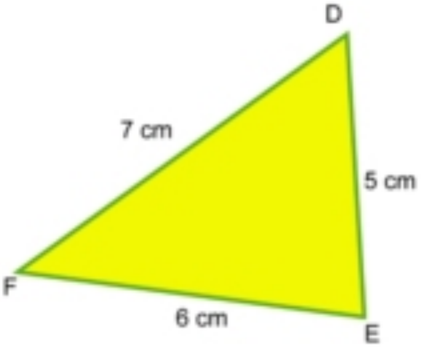
In the triangle ABC, if $\overline{AC}$ is the longest side and $(AB)^2 + (BC)^2 \neq (AC)^2$, then △ ABC is not a right triangle



Exercices (5 – 2)

1 **Complete and show which of the following triangles is a right angled triangle.**

A

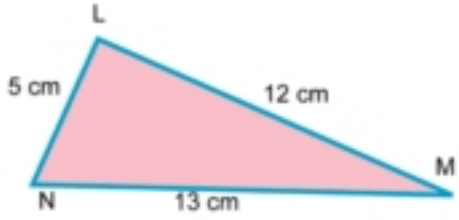


$DF^2 = ...$

$DE^2 + EF^2 = ...$

∴ the triangle is ....

B

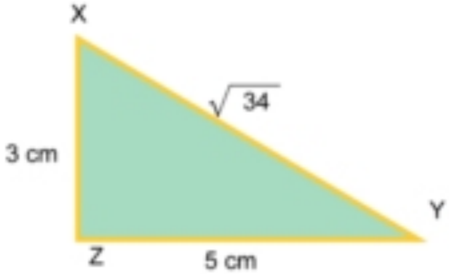


$MN^2 = ...$

$ML^2 + NL^2 = ...$

∴ the triangle is ....

C

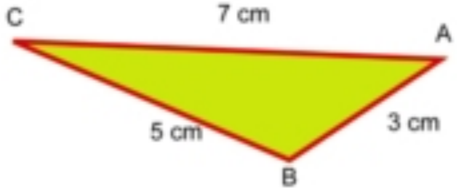


$XY^2 = (\sqrt{34})^2 = ...$

$YZ^2 + ZX^2 = ...$

∴ the triangle is ....

D



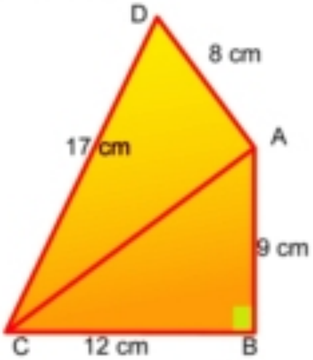
$AC^2 = ...$

$AB^2 + BC^2 = ...$

∴ the triangle is ....

2 **In the opposite figure**:ABCD is a quadrilateral $m(\measuredangle B) = 90^\circ$, $AB = 9$ cm, $BC = 12$ cm, $CD = 17$ cm and $DA = 8$cm.

Prove that:
$m(\measuredangle DAC) = 90^\circ$ then find the area of the figure ABCD.



Lesson Three

Projections

Think and Discuss

You will learn

- To find the projection of a point on a line.
- To find the projection of a line segment on a line.
- To find the prjection of a ray on a line.
- To find the prjection of a line on a line.

Key-Terms

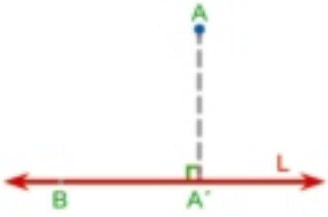
- Projection.
- Point.
- Line segment.
- Ray.
- Straight line.

A piece of chalk falls down on the earth:

Does it fall down vertically (perpendicular to the earth)? Does it leave a mark on the earth?

Projection of a point on a straight line

In the opposite figure:



L is a straight line, A and B are two points, where $A \notin L$ and $B \in L$.

Draw $\overline{AA'} \perp L$, where $A' \in L$.

The point A' (the point of intersection of $\overline{AA'}$ and L) **is called the projection of A on L.**

$\therefore B \in L$ $\therefore$ The projection of B on L is itself.

Note that:

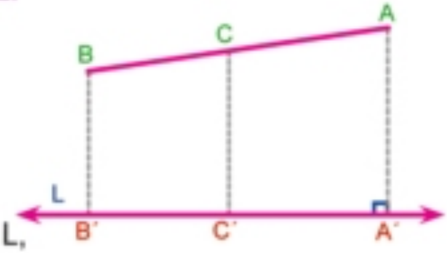
- **Projection of a point on a straight line is that the point of intersection of the perpendicular segment from this point and the straight line.**
- **If the point lies on the straight line, its projection on it is the same point.**

The Projection of a line Segment on a given straight line

Finding the projection of line segment $\overline{AB}$ on a line L.

If : A' is the projection of A on the straight line L
and B' is the projection of B on the straight line L,

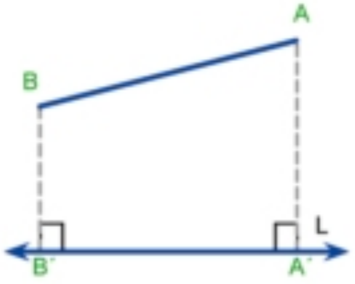
then $\overline{A'B'}$ is the projection of $\overline{AB}$ on L.



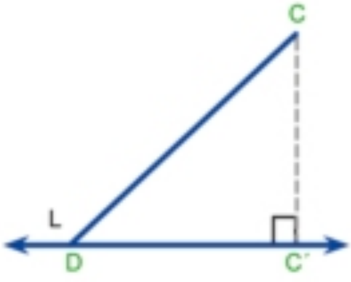
Note that If $C \in \overline{AB}$ and C' is its projection on L,
then $C' \in \overline{A'B'}$

Practice

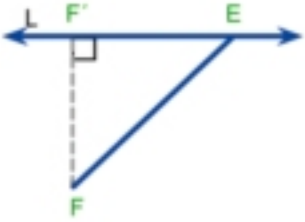
The following figures illustrate segments in different locations. Complete by writing down the projection of each one as shown in the first example:



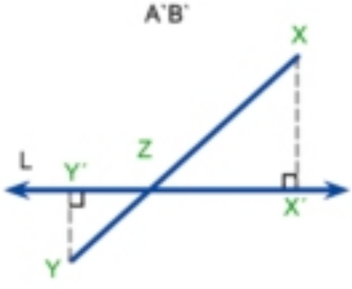
The projection of $\overline{AB}$ on L is $\overline{A'B'}$



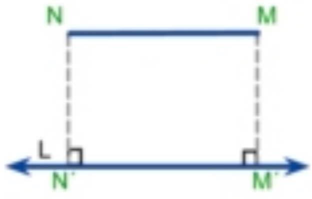
The projection of $\overline{CD}$ on L is ....



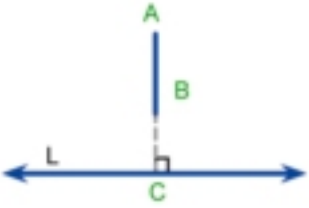
The projection of $\overline{EF}$ on L is ....



The projection of $\overline{XY}$ on L is ....



The projection of $\overline{MN}$ on L is ....



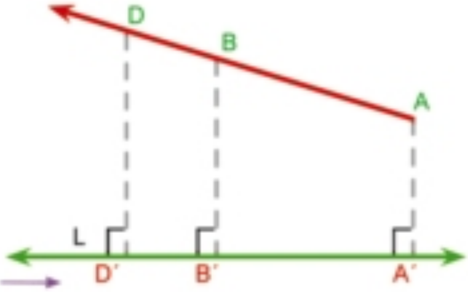
The projection of $\overline{AB}$ on L is ....

Note and Discuss:

- **A** The length of the projection of a line segment on a given line is less than or equal to the length of the segment itself.
- **B** When is the length of the projection of a line segment on a given line equal to the length of the segment itself?
- **C** When is the length of the projection of a segment on a given line equal to zero?

The Projection of a Ray on a straight line

Finding the projection of $\overrightarrow{AB}$ on L.



Note that A` is the projection of A on L
B` is the projection of B on L.

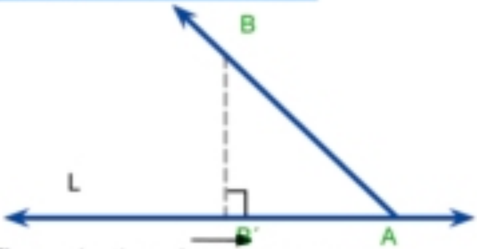
If D ∈ $\overrightarrow{AB}$, D ∉ $\overline{AB}$

and D` is the projection of D on L,

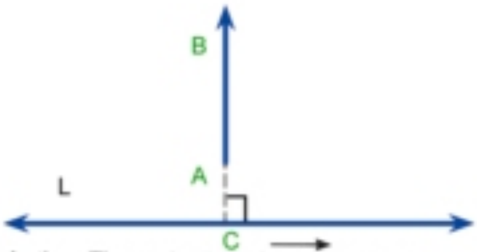
then D` ∈ $\overrightarrow{A`B`}$

∴ The projection of $\overrightarrow{AB}$ on L is $\overrightarrow{A`B`}$:

Observe and complete:



The projection of $\overrightarrow{AB}$ on L is..............

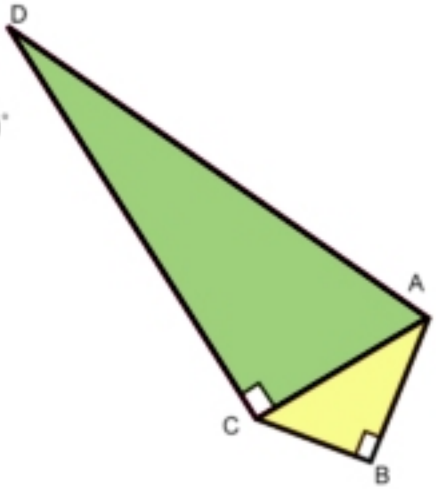


If $\overrightarrow{AB}$ ⊥ L, then The projection of $\overrightarrow{AB}$ on L is ....

Let's think

- **A** What is the projection of a line on a given line?
- **B** Can the projection of a line on a given line be a point?
- **C** Explain your answers by drawing different figures of a projection of a line on a given line and keep it in your portfolio file.

Practice:(1)

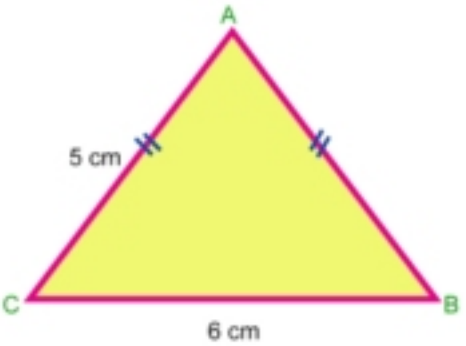


In the opposite figure: m (∠ B) = m (∠ ACD) = 90˚

Complete:

- **A** The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is ……………
- **B** The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is ……………
- **C** The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ……………

Practice:(2)



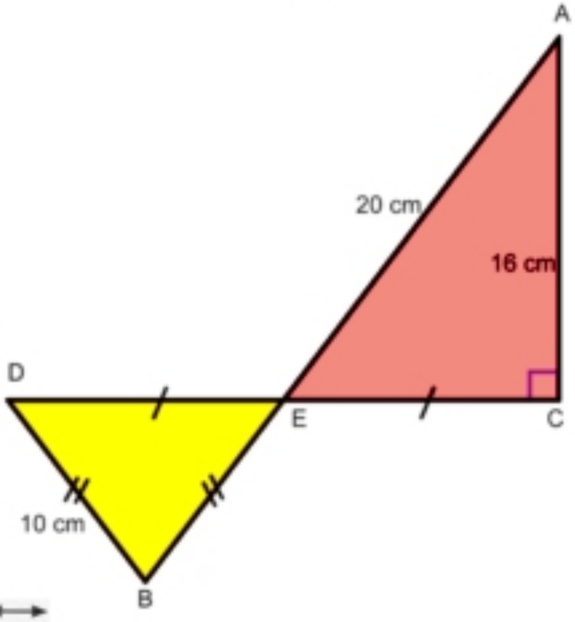
In the opposite figure:

ABC is a triangle, with AB = AC = 5cm,
and BC = 6cm.

Find:

- **A** The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$.
- **B** The area of the triangle ABC.

Practice:(3)



In the opposite figure:

$\overline{AB} \cap \overline{CD} = \{E\}$, E is the midpoint of $\overline{CD}$,

AC = 16cm, AE = 20cm,

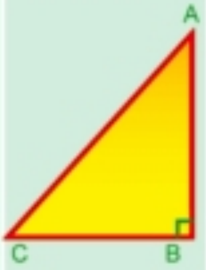
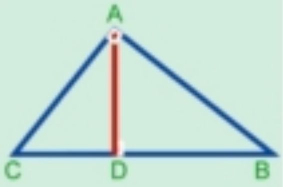
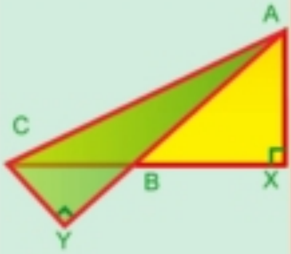
BD = B E = 10cm.

Find:

- **A** The length of the projection of $\overline{BD}$ on $\overleftrightarrow{CD}$
- **B** The length of the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$.

Exercices (5 – 3)

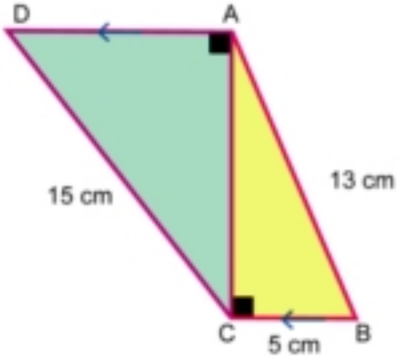
1 **Complete the following table:**

Figures / Projections			
Projection of $\overline{AC}$ sur $\overleftrightarrow{BC}$	....	....	$\overline{XC}$
Projection of $\overline{AB}$ sur $\overleftrightarrow{BC}$	....	....	....
Projection of $\overline{AC}$ sur $\overleftrightarrow{AB}$	....	....	....
Projection of $\overline{BC}$ sur $\overleftrightarrow{AB}$	....	....	....

2 **In the following figure:**

$\overline{AD} \parallel \overline{BC}$, $AB = 13$cm, $BC = 5$cm, $CD = 15$cm,

$m(\angle ACB) = m(\angle DAC) = 90^\circ$



Find: **A** The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$.

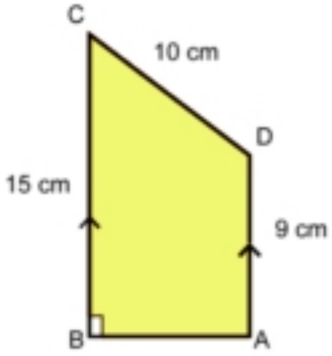
B The length of the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$.

3 **In the opposite figure:**

$ABCD$ is a trapezium, in which $\overline{AD} \parallel \overline{BC}$,

$m(\angle ABC) = 90^\circ$. If $AD = 9$cm,

$DC = 10$cm, and $CB = 15$cm.



Find: **A** The length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

B The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$

Euclidean Theorem

Lesson Four

Think and Discuss

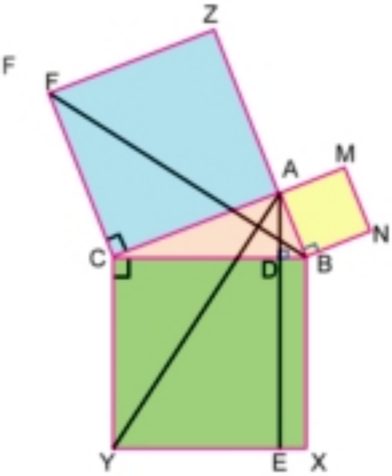
You will learn

- Euclidean Theorem.
- Applications on Euclidean theorem.

In the opposite figure:

❶ ABC is a right angled triangle at A. ABNM, ACFZ and BXYC are squares drawn on the sides of the triangle ABC.

❷ Draw $\overrightarrow{AD} \perp \overline{BC}$ and intersects it at D and intersects $\overline{XY}$ at E. Draw $\overline{BF}$ and $\overline{AY}$ as shown in the opposite figure.



Note that:

$$m(\angle BCF) = m(\angle YCA)$$

$\triangle BCF \equiv \triangle YCA$ **Why**? ?

area of $\triangle BCF = \frac{1}{2}$ the area of the square ACFZ **Why**?

area of $\triangle YCA = \frac{1}{2}$ the area of the rectangle EYCD **Why**?

Thus: the **area** of the square ACFZ = the **area** of the rectangle EYCD

$AC^2 = CD \times CY$ **Why**?

$\therefore AC^2 = CD \times CB$

= The length of the projection of $\overline{AC}$ × The length of the hypotenuse $\overline{BC}$

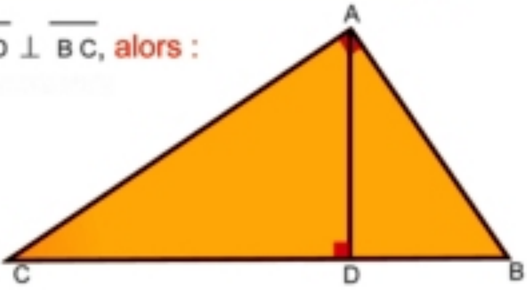
Euclidean Theory:

In the right-angled triangle, the area of the square on a side of the right angle is equal to the area of the rectangle which its dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse

i.e. ABC is a right angled triangle at A and $\overline{AD} \perp \overline{BC}$, alors :

$BA^2 = BD \times BC$

$CA^2 = CD \times CB$



Corollary:

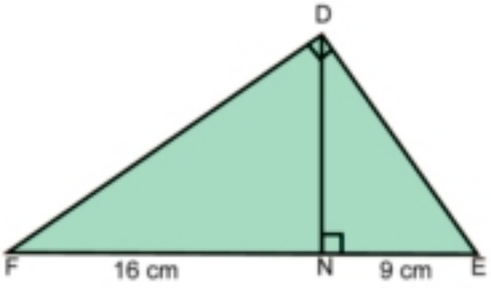
$$\boxed{(AD)^2 = DB \times DC}$$

Practice

In the opposite figure:

$\triangle$ DEF is a right angled triangle at D, $\overline{DN} \perp \overline{EF}$,

EN = 9cm and NF = 16cm



Complete:

$(DE)^2$ = EN × EF (Euclidean Theorem)

= .... × .... $\therefore$ DE = ...cm

$(DF)^2$ = FN × ...... (Euclidean Theorem)

= .... × .... $\therefore$ DF = ...cm

$(DN)^2$ = NE × NF. (...........)

= .... × .... $\therefore$ DN = ...cm

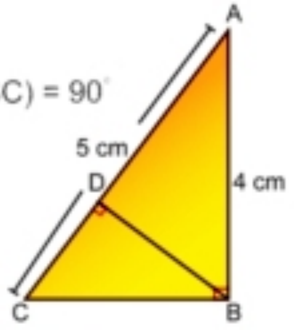
Let's think

Is DN × EF = DE × DF ? Why?

Exercices (5 – 4)

1 **In the opposite figure:** ABC is a triangle in which, $m(\angle ABC) = 90^\circ$

AB = 4cm, AC = 5cm and $\overline{BD} \perp \overline{AC}$. **Complete**



A BC = ... cm

B AD = ... cm

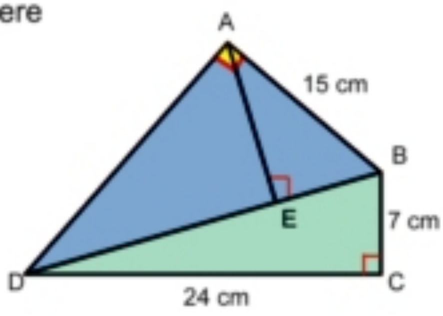
C BD = ... cm

D Area Δ DBC = ... cm^2

2 **In the opposite figure**: ABCD is a quadrilateral, where $m(\angle BCD) = m(\angle BAD) = 90^\circ$,

$\overline{AE} \perp \overline{BD}$, BC = 7 cm, CD = 24 cm et AB = 15 cm

Find:



A The length of $\overline{BD}$ and $\overline{AD}$

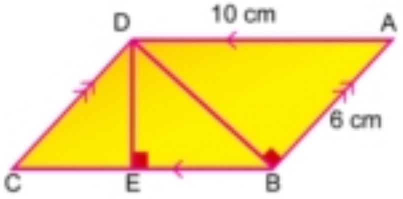
B The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$

C The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AE}$

3 **In the opposite figure:** ABCD is a parallelogram,

AB = 6cm, AD = 10cm, $\overline{DB} \perp \overline{AB}$.

Draw $\overline{DE} \perp \overline{BC}$ **find:**



A Area of the parallelogram ABCD.

B The length of the projection $\overline{DB}$ on $\overleftrightarrow{BC}$.

C The length of $\overline{DE}$

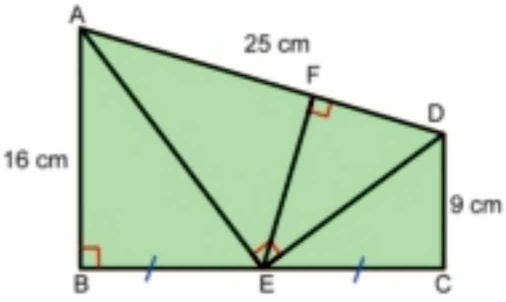
4 **In the opposite figure: ABCD is a trapezium with**

$\overline{AB} \parallel \overline{DC}$, $m(\angle ABC) = 90^\circ$,

E is the midpoint of $\overline{BC}$, AB = 16cm,

AD = 25cm, DC = 9cm, $\overline{AE} \perp \overline{ED}$

and $\overline{EF} \perp \overline{AD}$



Find: **A** area of the trapezium ABCD.

B The length of the projection of $\overline{AE}$ on $\overleftrightarrow{AD}$.

Lesson Five

Classifying of Triangles according to their Angles

Think and Discuss

You will learn

- to determine the type of a triangle according to its angles

Key-Terms

- A right angled triangle.
- An acute angled triangle
- An obtuse angled triangle.

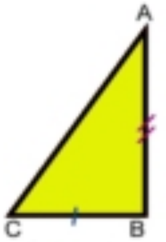


Fig. (1)

∠B is a right angle

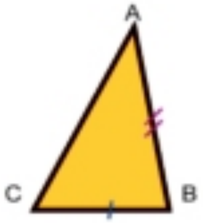


Fig. (2)

∠ B is an acute angle

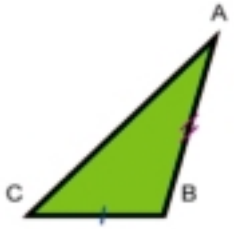


Fig. (3)

∠ B is an obtuse angle

Note that: **The length of $\overline{AB}$, does not change in all figures.**

The length of $\overline{BC}$, **does not change in all figures, Does it mean the length of** $\overline{AC}$ changes according to the opposite angle?.

Complete by writing **>** , **or = or <** :

in Fig. (1) $\therefore m(\angle B) = 90^\circ$ $\therefore (AB)^2 + (BC)^2$ .... $(AC)^2$

in Fig. (2) $\therefore m(\angle B) < 90^\circ$ $\therefore (AB)^2 + (BC)^2$ .... $(AC)^2$

in Fig. (3) $\therefore m(\angle B) > 90^\circ$ $\therefore (AB)^2 + (BC)^2$ .... $(AC)^2$

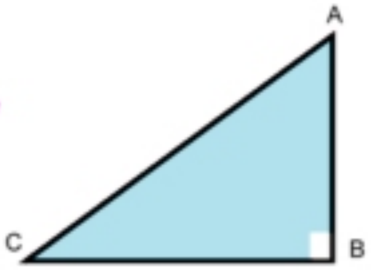
when is m ($\angle$ B)= 90°?:

Determining the type of triangle according to its angles, in case of knowing the lengths of its three sides.

We compare the square length of the longest side of the triangle and the sum of squares of the other two sides

I: If the

square length of the longest side is equal to the sum of the squares lengths of the other two sides, then the triangle is a right angled triangle.

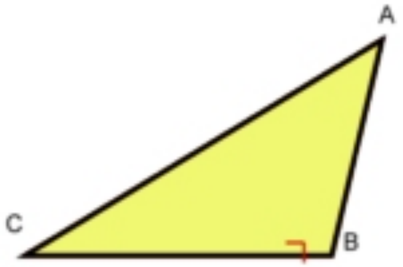


In △ ABC: $(AC)^2 = (AB)^2 + (BC)^2$

∴ ∠ **B is a right angle.**

II: If the

square length of the longest side > sum of squares lengths of the other two sides, then the triangle is an obtuse angled triangle.

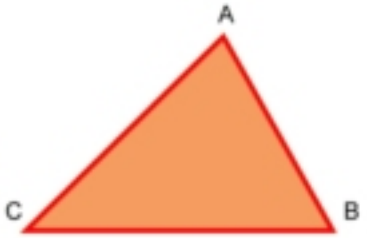


In △ ABC: $(AC)^2 > (AB)^2 + (BC)^2$

∴ ∠ **B is an obtuse angle.**

III: If the

square length of the longest side < the sum of squares lengths of the two other sides, then the triangle is an acute triangle.



In △ ABC: $(AC)^2 < (AB)^2 + (BC)^2$

∴ ∠B is an acute angle why?

Exemple :

Determine the type of the angle which has the greatest measure in △ ABC, where

AB = 8cm , BC = 10cm and CA = 7cm

What is the type of the triangle according to its angles?

Solution

∵ The greatest angle is opposite to the longest side.

∴ ∠A is the greatest angle in △ ABC, since $\overline{BC}$ is the longest side. $(BC)^2 = (10)^2 = 100$

$$AB^2 + AC^2 = (8)^2 + (7)^2$$
$$= 64 + 49 = 113$$

$\because (BC)^2 < (AB)^2 + (AC)^2$ $\therefore \angle A$ is an acute angle

$\because \angle A$ is the greatest angle

$\therefore \triangle ABC$ is an acute angled triangle

Exercices (5 – 5)

Determine in △ ABC the type of ∠ A :

A AB = 8 cm , BC = 10 cm , AC = 6 cm
B AB = 12 cm , BC = 13 cm , AC = 7 cm
C AB = 3 cm , BC = 7 cm , AC = 5 cm

General Exercices

1 **Determine the type of the greatest angle in** △ ABC, where:

A AB = 9 cm , BC = 10 cm , AC = 12 cm
B AB = 5 cm , BC = 12 cm , AC = 13 cm
C AB = 7 cm , BC = 16 cm , AC = 14cm

Determine the type of the triangle according to its angles.

2 **In the opposite figure:**

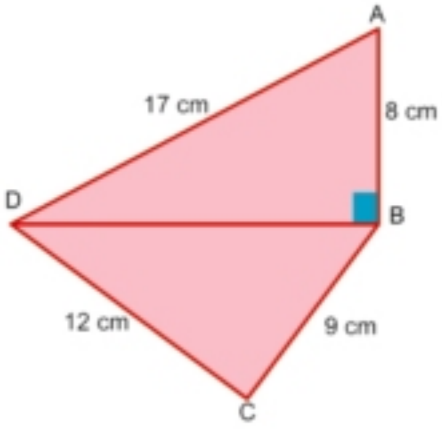
ABCD is a quadrilateral in which

AB = 8cm, BC = 9cm,

CD = 12cm, AD = 17cm and $\overline{DB} \perp \overline{AB}$

A Find the length of the projection of $\overline{AD}$ on $\overleftrightarrow{BD}$

B Determine the type of △ BCD according to its angles.



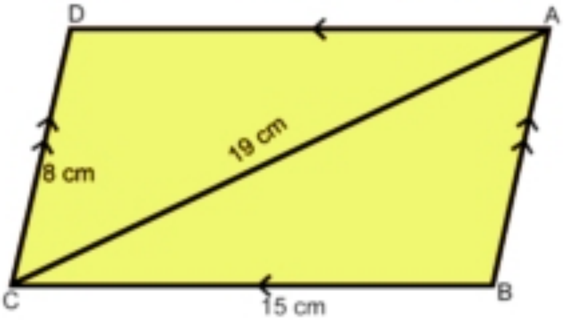
③ **In the opposite figure:**

ABCD is a parallelogram with

BC = 15cm, CD = 8cm, and

AC = 19cm.

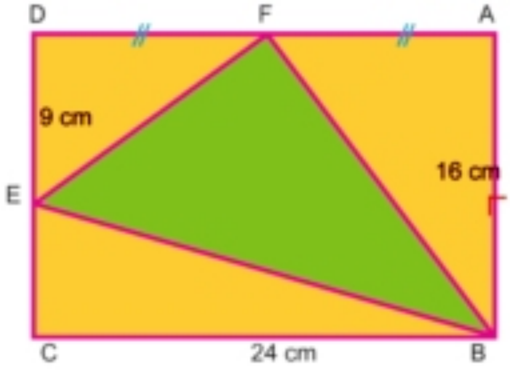
Prove that: ∠ ABC is an obtuse angle.



④ Find the length of $\overline{BC}$ in △ **ABC , in which**: $(AB)^2 > (AC)^2 + (BC)^2$, AB = 15cm, AC = 13cm, $\overline{AD} \perp \overleftrightarrow{BC}$ and intersects it at D, AD = 12cm.

⑤ **In the opposite figure:**

ABCD is a rectangle in which AB = 16cm, BC = 24cm, E ∈ $\overline{CD}$,and DE = 9cm. Classify the triangle BFE according to the measures of its angles.



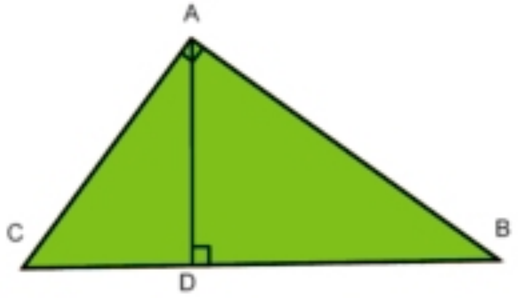
⑥ **In the opposite figure:**

△ ABC, in which m (∠ BAC) = 90˚,

$\overline{AD} \perp \overline{BC}$, AB = 8cm, and

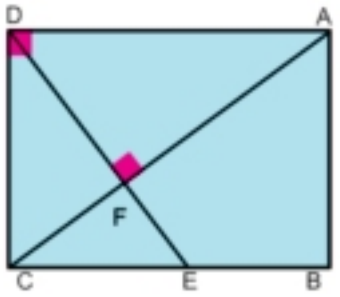
AC = 6cm.

Find BD, CD and AD.



⑦ ABCD is a rectangle in which AB = 30 cm, AD = 40cm, $\overrightarrow{DE} \perp \overline{AC}$ and intersects $\overline{AC}$ at F and intersects $\overline{BC}$ **in E.**

Find the length of $\overline{AF}$, $\overline{DF}$ and $\overline{EC}$.



Activites

In the opposite figure:

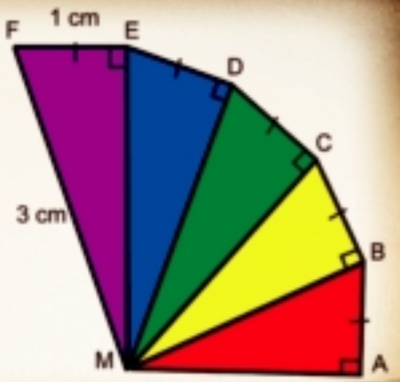
AB = BC = CD = DE = EF = 1 cm and

MF = 3cm

Find:

A The length of the projection of $\overline{FM}$ on $\overleftrightarrow{EM}$

B The length of the projection of $\overline{BM}$ on $\overleftrightarrow{AM}$



Unit test

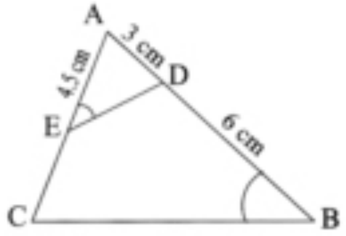
(1) In the opposite figure :

m(∠ AED) = m (∠ B)

, AD = 3 cm, AE = 4.5 cm، BD = 6 cm.

First : Prove that Δ ADE ~ Δ ACB.

Second : Find the length of $\overline{CE}$.



(2) In the opposite figure :

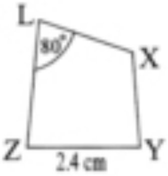
If the figure ABCD ~ the figure XZYL,

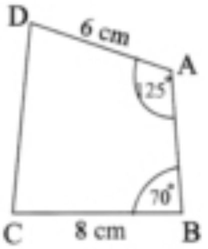
(a) Calculate m (∠ BCD)

(b) Calculate the length of $\overline{XL}$,

determine the ratio of the enlargement.

(c) If the perimeter of the figure ABCD = 26cm, what is the perimeter of the figure XYZL?





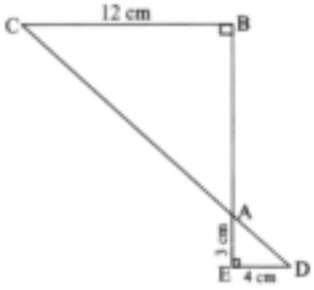
(3) In the opposite figure :

$\overline{BE} \cap \overline{DC}$ = { A } ,

m (∠ B) = m (∠ E) = 90°

(a) Prove that : Δ ABC ~ Δ AED

(b) Find the lebgth of $\overline{BE}$, $\overline{AC}$

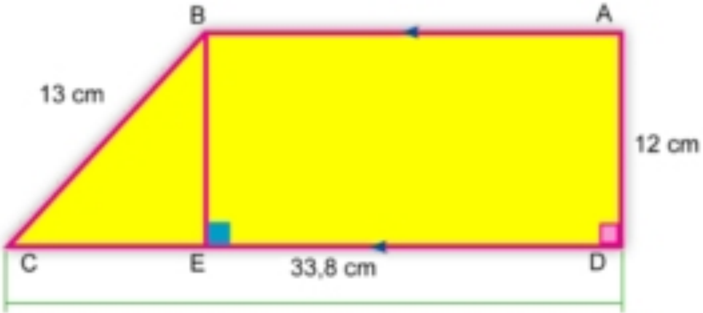


4 **In the opposite figure: ABCD is a trapezium in which**

$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \perp \overline{DC}$,

AD = 12cm, BC = 13cm,

DC =33.8cm, and $\overline{BE} \perp \overline{DC}$



I: Find

- A The length of $\overline{CE}$, $\overline{AB}$
- B The length of $\overline{DB}$
- C The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$
- D The area of the trapezium ABCD.

II: Prove that: m (∠ DBC) = 90°

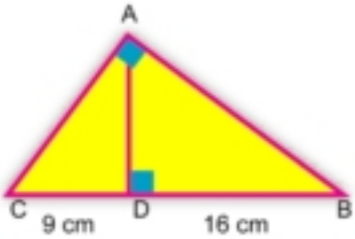
5 **In the opposite figure:**

△ ABC is a right angled triangle at A in which

$\overline{AD} \perp \overline{BC}$, BD = 16cm, CD = 9cm.

Find the lengths of $\overline{AB}$, $\overline{AC}$ **and** $\overline{AD}$

and calculate the area of △ ABC



Model (1)

[1] complete the following

(1) If $2^{x+3} = 1$, then $x =$..............

(2) If $X + Y = 4$, $X - Y = 2$ then $X^2 - Y^2$ equals .....

(3) The solution set of the equation $X^2 - 1 = 8$ where $X \in Z^+$ is ....

(4) If $2^X = 3$ then $8^{-X} =$.....

(5) $1 - \frac{3}{4} =$..............%

[2] Choose the correct answer

(1) $\frac{5^{-2} \times \sqrt{5}}{5\sqrt{5}} =$...................... ($\frac{1}{125}$, $\frac{1}{25}$, 25 , 125)

(2) $Z - Z^- =$...................... (Z^+, N , ϕ , {o})

(3) The volume of a cube of side length 3cm = ... cm^3

(9 , 12 , 27 , 81)

(4) The expression $X^2 + KX + 36$ is a perfect square when K equals ...

(±6 ، ±8 ، ±12 ، ±18)

(5) A regular die is thrown and observed the upper face, then the probability of appearance number divisible by 3 is

($\frac{1}{4}$ ، $\frac{1}{3}$ ، $\frac{1}{2}$ ، $\frac{3}{4}$)

(6) If $(\frac{5}{3})^x = \frac{27}{125}$ then $x =$.................. (-5 , -3 , 3 , 5)

[3] Factorize each of the following expressions

(a) $X^2 + 8X + 15$

(b) $2X^2 + 7X + 3$

(c) $X^3 - 1$

(d) $aX - 7a + 3X - 21$

[4] (a)Simplify to the simplest form $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

(b)Find the S.S for the following equation where $X \in R$

$X^2 - 8X + 12 = 0$

[5] (a) A bag contains a number of similar balls , 5 of them are white and the rest are red if the probability of drawn a red ball is $\frac{2}{3}$ find the number of all the balls

(b) If $3^X = 27$, $4^{X+Y} = 1$ find the values of X , Y

Model (2)

[1] complete the following :-

(1) If $7^{x-1} = 3^{x-1}$ then $x =$ ....

(2) $X^3 - \ldots\ldots = (X-2)(\ldots\ldots + 2X + 4)$

(3) $(5X-2Y)(25X^2 + 10XY + 4Y^2) = \ldots\ldots\ldots$

(4) If $\frac{2x}{5} = 6$, then $x =$ ....

(5) A bag contains 9 card labeled by numbers from 1 to 9 a card is drawn randomly then the probability that the card carries an odd number is ....

[2] Choose the correct answer

(1) If $X^3Y^{-3} = 8$ then $\frac{Y}{X} = \ldots\ldots$

(8 ، $\frac{1}{8}$ ، $\frac{1}{2}$ ، 2)

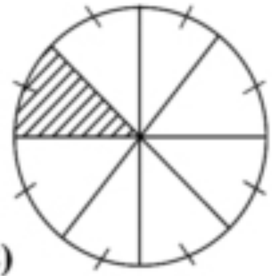
(2) The expression $X^2 + 4X + a$ is a perfect square when a equals ...

(3 ، 4 ، 8 ، 16)

(3) The S.S of the equation $X^2 - X = 0$ is .....

($\{0\}$ ، Φ ، $\{0,1\}$ ، $\{1\}$)

(4) In the figure opposite the shaded region represents... the circle ($\frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$)



(5) If $3^x + 3^x + 3^x = 1$ then $x =$......... ($-1, 0, \frac{1}{3}, 1$)

(6) If $6^x = 11$ then $6^{x+1} =$ .... (12, 22, 66, 72)

[3] Factorize each of the following

(1) $4X^2 - 9$

(2) $X^3 + 8$

(3) $X^2 - 5X$

(4) $X^2 - X - 6$

[4] Find the S.S of the following equation in R : $X^2 - X - 6 = 0$

(b) Simplify to the simplest form : $\frac{\left(\sqrt{2}\right)^{5}\times(3)^{-2}}{3\times\left(\sqrt{2}\right)^{9}}$

[5] (a) If $\frac{2^{x}\times 3^{x}}{(12)^{x}}=\frac{1}{2}$. **Find the value of** x

(b) A bag contains a number of similar balls some of them are red , 2 green , 4 blue and the rest are red if the probability of drawn a ball with green color is $\frac{1}{6}$ find the number of red balls

Model (3)

“For integrated pupils”

"خاص بطلاب الدمج"

Questions (1) Choose the correct answer:

(1) The solution set in R of the equation $x^2 + 25 = 0$ is ....

(a) ± 5 (b) 5 (c) -5 (d) ϕ

(2) If the expression $x^2 + a x + 9$ is a perfect square, then a = ...

(a) 3 (b) 6 (c) 9 (d) 18

(3) If $(x - 1)$ is one factor of the expression $x^2 - 4x + 3$, then the other factor is ....

(a) x + 3 (b) x + 1 (c) x – 3 (d) x - y

(4) If $\left(\frac{5}{3}\right)^{x} = \left(\frac{3}{5}\right)^{2}$, then x = ...

(a) – 2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(5) The probability of the sure event = ...

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

Questions (2):

Join from the column (A) to the suitable in the column (B)

(A)
(1) If $a^2 - b^2 = 15$, $a + b = 3$, then $a - b = \dots\dots\dots$
(2) If one digit of the number 37450 is chosen at random, then the probability of the chosen number is an even = ...
(3) If $(x + 3y)^2 = x^2 + kxy + y^2$, then $k = \dots\dots\dots$
(4) $4^3 + 4^3 + 4^3 + 4^3 = \dots\dots\dots$
(5) The probability of the impossible event = ...

(B)
• 5
• 6
• $\frac{2}{5}$
• 0
• 4^4

Questions (3): **Complete:**

(1) $x^2 - y^2 = (\dots\dots\dots - \dots\dots\dots)(\dots\dots\dots + \dots\dots\dots)$

(2) $y^3 - 8 = (\dots\dots\dots - \dots\dots\dots)(x^2 + 2x + \dots\dots\dots)$

(3) $x^2 - 5x + 6 = \dots\dots\dots\ (x - \dots\dots\dots)(\dots\dots\dots - 3)$

(4) $(a + b)x + (a + b)y = (a + \dots\dots\dots)(\dots\dots\dots + \dots\dots\dots)$

Questions (4): **Put (√) or (x) :**

(1) A school has 320 pupils, the probability of the ideal buy is 0.6, then the number of girls = 120 ( )

(2) If $3^x = 27$, then $x = \frac{1}{3}$ ( )

(3) A card is drawn at random, from cards numbered from 1 to 10 ( )

(4) The positive real which if its square is added to three times, the result will be 28 is 4 ( )

(5) The solution of the equation $x(x - 3)(x + 5) = 0$ in R is [0 , 3 , -5] ( )

Questions (5):

Complete the solution in which the expression $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$ in its simplest form

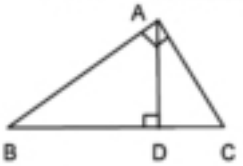
$$\frac{(2^{\dots})^n \times (\dots\dots \times 3)^{2n}}{2^{4n} \times 3^{2n}} = \frac{2^{\dots} \times \dots\dots^{2n} \times 3^{2n}}{2^{4n} \times 3^{2n}} \qquad = 2^{\dots + 2n \dots} \times 2^{2n\dots}$$

$$= 2^{\dots} \times 3^{\dots}$$

$$= \dots\dots\dots$$

Geometry
Model (1)

[1] complete the following



(1) In the figure opposite AB x ……… = BC x AD

(2) In Δ ABC if $(AC)^2 + (BC)^2 = (AB)^2$ then $m(\angle.....) = 90°$

(3) If the point $A \in$ the line L then the projection of the point A on the line L is .....

(4) The area of the circle of Diameter 14cm = ………… cm^2 ($\pi = \frac{22}{7}$)

(5) A trapezium whose bases length are 8cm , 10cm and its height 5cm then its area equals ……cm²

[2] choose the correct answer :

(1) in Δ ABC if $(AB)^2 > (BC)^2 + (AC)^2$ then angle C is …

(a) acute (b) right (c) obtuse (d) straight

(2) A rhomnus whose diagonals length are 6cm , 10cm has area ……cm²

(a) 60 (b)30 (c)15 (d)10

(3) The ratio between the lengths of two corresponding sides of two similar polygons is 3:5 then the ratio between their perimeters is

(a) 2:5 (b)5:3 (c)3:5 (d)1:2

(4) If the area of a trapezium is 100 cm² and its height is 5cm then the length of its middle base =….cm

(a) 20 (b)30 (c)40 (d)50

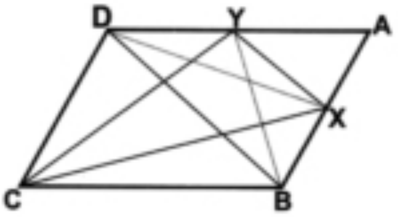
(5) ABCD is a parallelogram in which $m(\angle A) = 70°$, then

$m(\angle B) = ...°$ (70, 110, 180, 360)

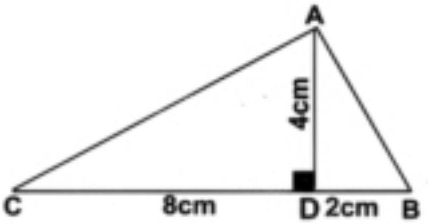
(6) Measure of each angle of the regular pentagon is = ...° (90, 108, 120, 540)

[3] (a) The sides lengths of one of two similar triangles are 3cm , 4cm , 5cm And the perimeter of the other triangle is 36 find the side lengths of the other triangle

(b) In the opposite figure
ABCD is a parallelogram
$X \in \overline{AB}$, $Y \in \overline{AD}$ such that
Area of ΔCBX = Area of ΔCYD
Prove that :
$\overleftrightarrow{XY} // \overleftrightarrow{BD}$



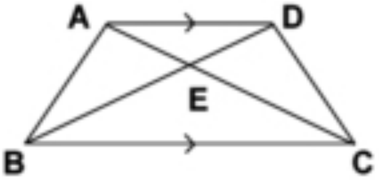
(4) (a) in the opposite figure
ABC triangle in which BD=2cm
, CD=8cm , AD=4cm , $\overline{AD} \perp \overline{BC}$
Prove that : $m(\angle BAC) = 90^\circ$



(b) ABCD is a parallelogram in which AB=18cm
, DE=15cm and BC=12cm we draw $\overline{DE} \perp \overline{BC}$, ,$\overline{DO} \perp \overline{AB}$
Calculate the area of ▱ ABCD and find the length of DO

(5) (a) ABC is a triangle in which $m(\angle A) = 50^\circ$, $m(\angle B) = 60^\circ$. Arrange the lengths of the sides of the triangle in descending order.

(b) In the figure opposite:
ABCD is a quadrilateral in which
$\overline{AD} \parallel \overline{BC}, \overline{AC} \cap \overline{BD} = \{E\}$
Prove that:
Area of Δ ABE = Area of Δ DCE



Model (2)

[1] complete the following

(1) The two polygons are similar if their corresponding sides are ......and their corresponding angles are.....

(2) The area of rhombus is $24cm^2$, the length of one of its diagonals 8cm then the length of th eother diagonal is.....

(3) A triangle whose side lengths 6cm , 8cm and 11cm then its type according to its angles is ....

(4) Area of triangle is equal to half of area of a parallelogram if they have a common ....

[2] choose the correct answer

(1) A trapezium whose bases length are 6cm , 8cm then the length of its middle base equals ....cm

(a) 48 (b) 24 (c)14 (d)7

(2) If two polygons are similar and the ratio between the lengths of two corresponding sides is 1:3 and the perimeter of smaller polygons is 15cm then the perimeter of the greater polygon is.....cm

(a)30 (b)45 (c)60 (d)75

(3) If the area of the triangle $24cm^2$ and its height =8cm then the length of the corresponding base ......

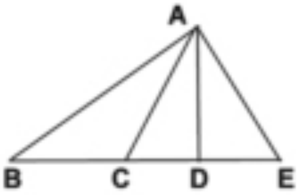
(a)16 (b)6 (c)3 (d) 2

(4) ABC is right angled triangle at B , $\overline{BD} \perp \overline{AC}$ then the projection of $\overline{BD}$ on $\overline{AC}$ is (a) {A} (b) {B} (c) {C} (d) {D}

(5) A square of perimeter 20cm then its area equals ....cm^2

(a)20 (b)25 (c)50 (d)100

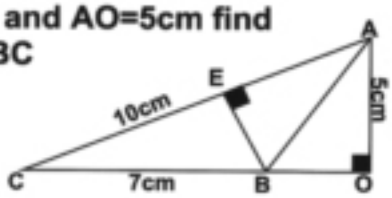
(6) The number of the trialngles in the opposite figure = ... (3, 4, 5, 6)



[3] in the opposite figure

$\overline{AO} \perp \overline{CB}$, $\overline{BE} \perp \overline{AC}$, AC=10cm , BC=7cm and AO=5cm find

(i) the length of $\overline{BE}$ (ii) area of ΔABC



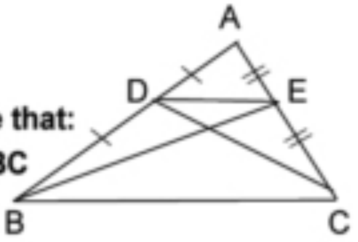
[4] (a) ABCD is a parallelogram in which AB=8cm , AC=20cm And BD=12cm prove that $m(\angle ABD) = 90^\circ$ then find the area of this parallelogram ABCD

(b) In the figure opposite:

ABC is a triangle in which D is the mid-point of $\overline{AB}$, E is the mid-point of $\overline{AC}$, prove that:

First: Area of the triangle DBC = Area of triangle EBC

Second: $\overline{DE} \parallel \overline{BC}$



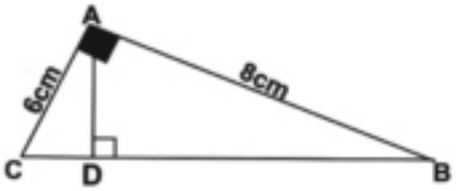
[5] (a) in the opposite figure ,

$\Delta DBA \sim \Delta ABC$, $m(\angle BAC) = 90^\circ$

Prove that : $\overrightarrow{AD} \perp \overrightarrow{BC}$

And if AD=8cm , AC=6cm

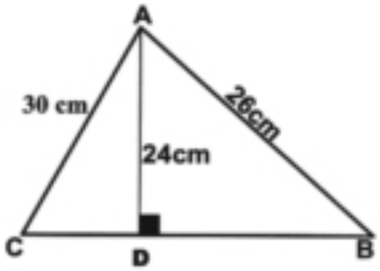
find the length of $\overline{BD}$



(b) In the opposite figure :

ABC is a triangle , $\overline{AD} \perp \overline{BC}$ if AD=24cm , AB=26cm And AC=30cm find BC

Then calculate area of ΔABC



Model (3)

"For integrated pupils"

"خاص بطلاب الدمج"

Questions (1) **Choose the correct answer:**

(1) The area of parallelogram whose length of its base 6cm and its corresponding height of this base 4 cm equals .... cm^2.

[12 , 20 , 24 , 48]

(2) The triangle whose lengths of its sides 6cm, 8 cm, 10 cm is ....

[Acute angled triangle, right angled triangle , obtuse angled triangle , otherwise]

(3) The rhombus whose lengths of its diagonals 6cm and 10 cm then its area = ... cm^2.

[60 , 30 , 15 10]

(4) Trapezium of length of its middle base 8 cm and surface area 56 cm^2 then its height = ... cm. [32 , 24 , 448 , 7]

(5) All .... similar.

[squares , triangles , rectangles , parallelograms]

Questions (2) **Complete each of the following:**

(1) The projection of point on a straight line is ....

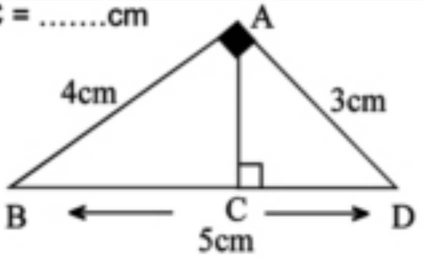
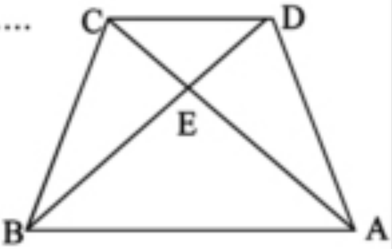
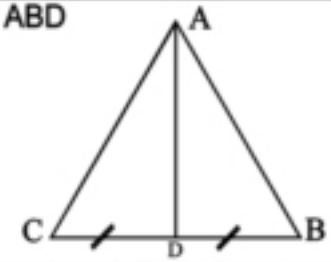
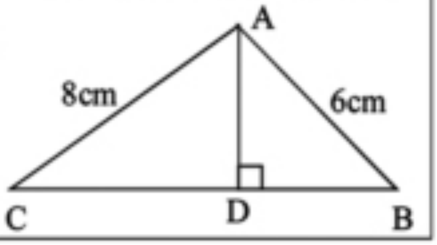
(2) If the triangle ABC is obtuse angled triangle at B then $(AC)^2$ ...... $(AB)^2 + (BC)^2$

(3) The square whose length of its diagonal 8 cm then its area = ... cm^2.

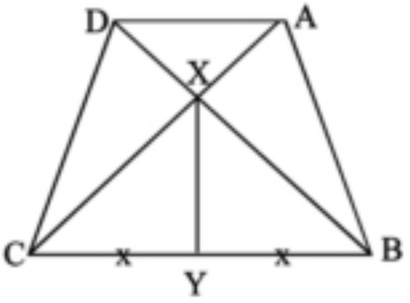
(4) The two triangles have same base and the vertices opposite to this base on straight line parallel to the base ....

(5) Area of triangle = $\frac{1}{2}$ x .... x corresponding height.

Questions (3) **Join from column (A) to suitable from column (B) :**

(A)	(B)
(1) In the opposite figure AC = ...cm 	• BEC
(2) In the opposite figure : Area of Δ AED = area of Δ .... 	• 2.4
(3) In the opposite figure area of Δ ABD = area of Δ .... 	• Congruent
(4) If the ratio of enlargement between two similar triangles = 1, then the two triangles	• 3.6
(5) In the opposite figure the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ = ... cm 	• ACD

Questions (4): **In the opposite figure:**



Area of the figure ABYX =

Area of the figure DCYX

Complete the proof:

To prove that $\overline{AD} // \overline{BC}$

Given:

A.T.P:

Proof: $\overline{xy}$ is median in Δ XBC

∴ area of Δ .... = area of Δ .... (1)

∵ area of the figure ABYX = area of the figure DCYX (2)

By subtracting (1) from (2)

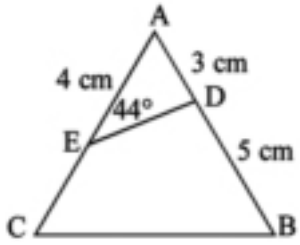
∴ Area of Δ .... = area of Δ ....

By adding area of Δ ADX to both side

∴ Area of Δ .... = area of Δ ....

∴ $\overline{AD} // \overline{BC}$

Questions (5): **In the opposite figure:**



Δ ABC ~ Δ AED

m(∠ AED) = 44°

AD = 3 cm, EA = 4 cm, DB = 5cm

BC = 8 cm Complete to find length of $\overline{ED}$ and $\overline{EC}$

Solution ∵ Δ ABC ~ Δ AED

$\therefore \frac{AB}{\text{........}} = \frac{\text{.......}}{ED} = \frac{CA}{DA}$ $\therefore \frac{8}{\text{.......}} = \frac{\text{.......}}{ED} = \frac{CA}{3}$

∴ ED = ... cm, AC = ... cm, EC = ...cm